A new Approach to Stochastic Simulation of EM Fields Using Universal Form of Polynomial Chaos Expansion Coefficients

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***Abstract*– In the paper we present a new method for simulation of EM fields distributions in propagation channel with random variables. W take advantage of universal closed form coefficients of polynomial chaos expansion for multiple stochastic variables case. The coefficients are the functions of parameters of probability distributions of a given propagation scenario features. This enables to obtain accurate results of stochastic EM wave distributions in relatively very short time when compared to the Monte Carlo or a general polynomial chaos expansion theory.**

1 INTRODUCTION

In the paper we will present a new method for stochastic simulations and analysis of electromagnetic (EM) fields distributions.

Electromagnetic fields simulations concern many subjects in areas of an antenna analysis, a propagation prediction in an open medium or in a waveguide. It is often important to include stochastic behavior of parameters of a considered scenario in these simulations when uncertainties of simulation input variables must taken into consideration.

Perhaps the most known method that enables to include stochastic behavior of parameters of a given simulation scenario is the Monte Carlo method. The advantage of this method is a high accuracy in determining stochastic distributions of output random variables but it requires a lot of iterations to obtain these accurate results, especially for the case of a standard deviation. Alternative method to the Monte Carlo algorithm is polynomial chaos expansion method [1]. This method enables to sufficiently decrease a time of stochastic simulations, but this time increase considerably with growing number of input stochastic variables of simulation scenario.

We present in the paper a new approach to stochastic analysis of EM field distributions. It bases on polynomial chaos expansion [1] but time consumptions related to application of our approach are a lot lower than for the case of a basic polynomial chaos expansion. The key advantage of our approach are universal expansion coefficients that are functions of parameters of stochastic distributions of simulation scenario variables (e.g. obstacle permittivity and conductivity). As in polynomial chaos theory, our universal expansion coefficients are required to calculate statistics of considered EM fields distributions.

Our universal coefficients include constants that can be tabulated separately for each wave phenomenon (e.g. reflection) and simulation scenario element (e.g. obstacle, antenna). The number of these constants for a single wave phenomenon and simulation scenario element depend on ranges of possible values of input stochastic variables that correspond to wave phenomenon and simulation scenario element. We use Jacobi polynomials othonormal basis for polynomial expansion for the best quality of approximation. We assume in our approach that all wave phenomena and simulation elements can be described by its own function (e.g. transfer function) as it is for the case of "ray tracing" algorithm. The paper is organized as follows. In Section II we will present our new method for stochastic simulations of EM fields distributions using our universal expansion coefficients. In Section III we will introduce a "ray tracing" simulation example which will be used to verify our new method against the Monte Carlo method. We will conclude the paper in Section IV.

2 UNIVERSAL EXPANSION COEFFICIENTS FOR MULTIPLE SIMULATION ELEMENTS

In this section we present the main points of our algorithm that enables the derivation of our universal expansion coefficients for a single simulation element (an antenna, a wave phenomenon or an obstacle) and multiple simulation elements. First we recall briefly the general polynomial chaos expansion theory [1] for the case of two stochastic variables in terms of stochastic analysis of EM field distributions. We use this theory the case of derivation of our universal expansion coefficients for a single simulation element.

When a propagation scenario is described by its transfer function, e.g. $T(\omega, \xi_1, \xi_2)$, where ξ_1 and ξ_2 are the stochastic variables which have probability distributions $p_1(\xi_1)$ and $p_2(\xi_2)$, respectively, a mean and a standard deviation of this stochastic transfer function for pulsation sample ω_n can be found as follows [1]:

$$\mu_{Tn} = a_{0,0,n} , \tag{1}$$

$$\sigma_{Tn} = \sqrt{\sum_{\substack{k_1,k_2 \\ k_1+k_2>0}} \gamma_{k_1}\gamma_{k_2} \cdot a_{k_1,k_2.n}{}^2} , \tag{2}$$

where:

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$$a_{k_1,k_2,n} = \frac{1}{\gamma_{k_1}\gamma_{k_2}} \times \int_{a_{\xi_1}}^{b_{\xi_1}}\int_{a_{\xi_2}}^{b_{\xi_2}} T(\omega_n,\xi_1,\xi_2)\varphi_{k_1}(\xi_1)\varphi_{k_2}(\xi_2)p_1(\xi_1)p_2(\xi_2)d\xi_2 d\xi_1 \quad (3)$$

while γ_{k1}, γ_{k2} are the normalization factors [1] and $\varphi_{k1}(\xi_1)$, $\varphi_{k2}(\xi_2)$ are chaotic polynomials that are orthonormal in domains limits $a_{\xi 1} \le \xi_1 \le b_{\xi 1}$, $a_{\xi 2} \le \xi_2 \le b_{\xi 2}$ for weighting functions $p_1(\xi_1)$, $p_2(\xi_2)$. Now we pass to the procedure for derivation of our universal expansion coefficients for the case of two stochastic variables.

In the first step we expand transfer function $T(\omega, \xi_1, \xi_2)$ for a given pulsation and stochastic variables domains limits that are sufficient to cover all possible values of ξ_1 and ξ_2 that we may need to deal with. We use for the expansion a Jacobi polynomial orthonormal basis with Beta distribution weighting functions [1]. We choose this basis because it is the most convenient for approximation of transfer functions with limited domains of stochastic variables. The expansion takes the form:

$$T(\omega_n,\xi_1,\xi_2) = \sum_{k_1,k_2} a_{k_1,k_2,n} P_{k_1}^{\alpha 0,\beta 0}(f_1(\xi_1)) P_{k_2}^{\alpha 0,\beta 0}(f_1(\xi_2)), \quad (4)$$

where:

$$f_1(\xi_{1/2}) = \xi_{1/2}\frac{2}{b_{\xi_{1/2}} - a_{\xi_{1/2}}} - \frac{b_{\xi_{1/2}} + a_{\xi_{1/2}}}{b_{\xi_{1/2}} - a_{\xi_{1/2}}}, \quad (5)$$

and $P_k^{\alpha,\beta}(\xi)$ is a Jacobi polynomial of *k-th* order.

The most time-consuming numerical calculations that are required to obtain coefficients from (3), are performed for freely chosen parameters of orthonormal basis and only once for a given propagation scenario. Then constant coefficients $a_{k1,k2,n}$ are tabulated and are used to derive our universal expansion coefficients. For presentation of this derivation we can assume now, that stochastic variables ξ_1 and ξ_2 have Gaussian probability distribution with parameters μ_1, σ_1 and μ_2, σ_2, respectively. Then our universal coefficients for a single simulation element can be derived from the following formula [1]:

$$b_{m_1,m_2,,n} = \frac{1}{\gamma_{m_1}\gamma_{m_2}} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T(\omega_n, f_2(\xi_1), f_2(\xi_1)) H_{m_1}(\xi_1) H_{m2}(\xi_2) \frac{e^{\frac{-\xi_1^2}{2}}}{\sqrt{2\pi}}\frac{e^{\frac{-\xi_2^2}{2}}}{\sqrt{2\pi}} d\xi_2 d\xi_1 \quad (6)$$

where:

$$f_2(\xi_{1/2}) = \sigma_{1/2}\xi_{1/2} + \mu_{1/2}, \quad (7)$$

and $H_k(x)$ is a Hermite polynomial of order k.

In the second step of our algorithm we substitute (4) into (6) and perform transformations of domains and we obtain:

$$b_{m_1,m_2,n} = \frac{1}{m_1! m_2!} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sum_{k_1,k_2} a_{k_1,k_2,n} P_{k_1,k_2}^{g,h,}(\xi_1,\xi_2) H_{m_1}(\xi_1) H_{m_2}(\xi_2) \frac{e^{\frac{-\xi_1^2}{2}} e^{\frac{-\xi_2^2}{2}}}{2\pi} d\xi_2 d\xi_1 \quad (8)$$

where:

$$P_{k_1,k_2}^{g,h,}(\xi_1,\xi_2) = P_{k_1}^{\alpha 0,\beta 0}(g_1\xi_1 + h_1) P_{k_2}^{\alpha 0,\beta 0}(g_2\xi_2 + h_2) \quad (9)$$

$$g_{1/2} = \frac{2\sigma_{1/2}}{b_{\xi_{1/2}} - a_{\xi_{1/2}}}, \quad (10)$$

$$h_{1/2} = \frac{2\mu_{1/2} - b_{\xi_{1/2}} - a_{\xi_{1/2}}}{b_{\xi_{1/2}} - a_{\xi_{1/2}}}. \quad (11)$$

In the third step of our algorithm a Jacobi polynomial of *k-th* order is transformed into a sum of Hermite polynomials of maximum order k [3]. After grouping an accumulating the polynomial coefficients we get the following form of (8):

$$b_{m_1,m_2,n} = \frac{1}{m_1! m_2!} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sum_{k_1,k_2} d_{k_1,k_2,n} H_{k_1,k_2}^{g,h,}(\xi_1,\xi_2) H_{m_1}(\xi_1) H_{m_2}(\xi_2) \frac{e^{\frac{-\xi_1^2}{2}} e^{\frac{-\xi_2^2}{2}}}{2\pi} d\xi_2 d\xi_1 \quad (12)$$

where:

$$H_{k_1,k_2}^{g,h,}(\xi_1,\xi_2) = H_{k_1}(g_1\xi_1 + h_1) H_{k_2}(g_2\xi_2 + h_2) \quad (13)$$

$$d_{k_1,k_2,n} = \sum_{i_1 \ge k_1, i_2 \ge k_2} a_{i_1,i_2,n} c_{k_1}^{i_1} c_{k_2}^{i_2}, \quad (14)$$

where c_k^i is the weight of Jacobi polynomial of order *i-th* that compose Hermite polynomial of order *k-th*.

In the fourth step of the algorithm in order to calculate (12) analytically we use the following identity [4]:

$$H_k(x+y) = \sum_{j=0}^{k} \binom{k}{j} \cdot y^{k-j} \cdot H_j(x) \cdot \quad (15)$$

When (15) is applied in (13) we obtain the final form of our universal expansion coefficients in frequency domain:

$$b_{m_1,m_2,n} = \frac{1}{m_1! m_2!} \sum_{k_1,k_2} d_{k_1,k_2,n} G_{m_1,k_1}^{g_1,h_1} G_{m_2,k_2}^{g_2,h_2} \quad (16)$$

where:

$$G_{m_{1/2},k_{1/2}}^{g_{1/2},h_{1/2}} = \frac{h_{1/2}^{\ k_{1/2}}}{\left(\sqrt{1 - g_{1/2}^{\ -2}}\right)^{m_{1/2}}} \sum_{j=0}^{k_{1/2}} \left(\sqrt{\frac{g_{1/2}^{\ 2} - 1}{h_{1/2}^{\ 2}}}\right)^{j} Q(j, k_{1/2}, m_{1/2}), \quad (17)$$

where $Q(j, k, m)$ do not depend on μ or σ therefore can be tabulated as follows. When $(j - m) = 0, 2, 4, 6 \ldots$, we have:

$$Q(j,k_{1/2},m_{1/2}) = \frac{\frac{k_{1/2}!}{(k_{1/2}-j)!}}{\left(\frac{j-m_{1/2}}{2}\right)!\left(\sqrt{2}\right)^{j-m_{1/2}}}. \qquad (18)$$

while (18) is 0 for the rest values of $(j - m)$.It can be shown that the universal expansion coefficients for the case of more than two, e.g. W input stochastic variables of the transfer function of a single simulation element, expanded in the first step of our algorithm, has the form:

$$b_{[m],n} = \frac{1}{\gamma_{[m]}} \sum_{[k]} d_{[k],n} \prod_{w=1}^{W} G_{m_w,k_w}^{g_w,h_w} \qquad (19)$$

where $[m]$ and $[k]$ are multi-indexes [1], while $G_{m_w,k_w}^{g_w,h_w}$ can be calculated according to (17) when w-th stochastic variable has Gauss probabilistic distribution or using formula:

$$G_{m_w,k_w}^{g_w,h_w} = \sum_{j=0}^{k_w} \binom{k_w}{j} H_j\left(h(c_{\xi_w},d_{\xi_w})\right) g(c_{\xi_w},d_{\xi_w})^{k_w-j} I(\alpha,\beta), \qquad (20)$$

where $h(c_{\xi w}, d_{\xi w})$, $g(c_{\xi w}, d_{\xi w})$ and $I(\alpha, \beta)$ can be calculated from (12), (13) and (19-20) in [2], respectively when w-th stochastic variable has a beta probabilistic distribution which can model an asymmetric as well as an uniform distribution.

In order to find the universal expansion coefficients for the case of multiple simulation elements w take advantage universal expansion coefficients given by (19). It can be done when e.g. a transfer function of a multiple simulation elements is a product of transfer functions of separate simulation elements. Let us consider the two simulation elements case whose transfer function can be described by:

$$T(\omega_n,\xi_1,\xi_2,\xi_3,\xi_4) \sim T_1(\omega_n,\xi_1,\xi_2)\cdot T_2(\omega_n,\xi_3,\xi_4), \qquad (21)$$

Such a feature is met when "ray tracing" techniques are used for EM field propagation analysis. Transfer functions $T_1(\omega, \xi_1, \xi_2)$, $T_2(\omega, \xi_3, \xi_4)$ can then describe e.g. reflection coefficients of two consecutive surfaces that occur on a way of a ray. The permittivities and conductivities of these surfaces are random variables and named by ξ_1 and ξ_2, respectively for the first surface and ξ_3 and ξ_4, for the second surface. The formulas for such reflection coefficients are well known in the literature. Consequently $d_{[k],n}$ constants in (19) can be calculated once and then tabulated for use in "ray tracing" simulator. The universal expansion coefficients of $T(\omega, \xi_1, \xi_2, \xi_3, \xi_4)$ can be written as follows:

$$B_{[M],n} = b_{[m1],n} \cdot b_{[m2],n} \qquad (22)$$

where $b_{[m1],n}$ and $b_{[m2],n}$ are universal expansion coefficients of $T_1(\omega, \xi_1, \xi_2)$ and $T_2(\omega, \xi_3, \xi_4)$, respectively while multi-index $[M]$ is a concatenation of multi-indexes $[m1]$ and $[m2]$.

3 SIMULATION EXAMPLE

As a simulation example for the results presented in the previous section we consider an indoor "ray tracing" simulation scenario shown in Fig. 1.

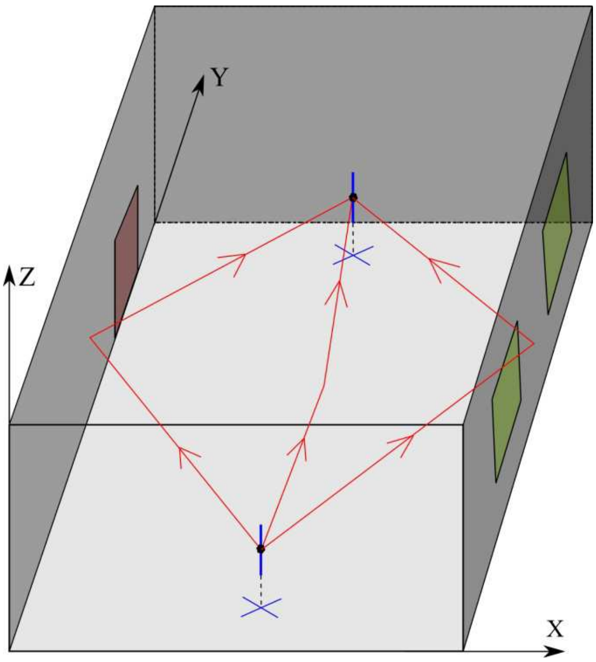

Figure 1: Indoor "ray tracing" simulation scenario.

The width (x dimension) and height (z dimension) of the room in Fig. 1. is 10m, while the length (y dimension) of the room is 20m. The z, y and z coordinates of the window corners are (10m, 6m, 1.5m), (10m, 8m, 2.5m) and (10m, 13m, 1.5m), (10m, 15m, 2.5m). The corresponding coordinated of doors are (0m, 15m, 0m), (0m, 16m, 2m). The transmitting and receiving antennas are half-wavelength dipoles polarized vertically. The transmitting antenna phase center is positioned at coordinates x = 5m, z = 2m and y = 3m. The corresponding coordinates for the receiving antenna move along line x = 5m, z = 2m from y = 5m to y = 17m. The relative permittivities and conductivities of 4 walls, floor, ceiling, windows and doors are independent random variables with uniform probability density functions with mean and standard deviations given in a table below:

Obstacle	ε_r		σ [s/m]	
	mean	st. dev.	mean	st. dev.
Wall	3	0.5	1	0.15
Floor	6	1	1	0.15
Ceiling	6	1	1	0.15
Window	6.5	1	0.15	0.03
Doors	3	0.5	0.03	0.005

Table 1: Parameters of probability density functions of materials composing room considered in Fig. 1.

We simulated the described above scenario using our universal expansion coefficients calculated according to (22) and using Monte Carlo method as a reference. The simulation results present the received signal power in dB scale along the distance between transmitting and receiving antennas when the power of a transmitted signal is 0dBm while frequency is 1.8GHz. The results of a mean given in dBm and a standard deviation given in dB of the signal power at the matched receiver antenna is shown in Fig. 2 and Fig. 3, respectively. The curves representing our universal expansion coefficients are given by dashed line while Monte Carlo results are represented by circle sign curves.

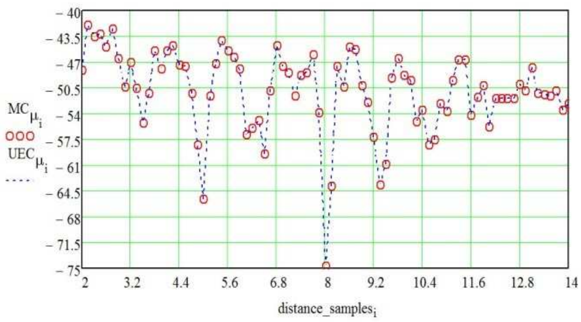

Figure 2: The results of mean of the power at receiving antenna given in dBm for antenna separations 2m – 14m.

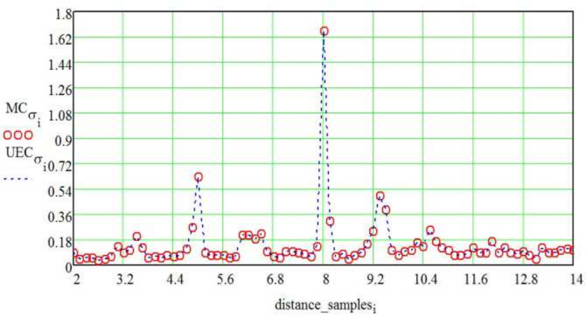

Figure 3: The results of standard deviation of the power at receiving antenna given in dB for antenna separations 2m – 14m.

It can be seen from the above figures that the results obtained with our universal expansion coefficients are very accurate and agree very well with the results of Monte Carlo method. The latter results were obtained in over 3000 longer time.

4 CONCLUSIONS

We have presented in this paper new method for stochastic simulations of EM fields distributions. The method takes advantage of our universal expansion coefficients of single simulation elements. The values of constants that occur in our universal expansion coefficients for single simulation elements relate to a given antenna, wave phenomenon and an obstacle, therefore we can say that they carry an information about the simulation element. The constants have to be calculated only once and then can be used to calculate universal expansion coefficients related to multiple simulation elements during the "ray tracing" simulation process. We verified our results against the Monte Carlo method. For space saving issues only one simulation example were shown. We can conclude that the stochastic results of simulations that use our expansion coefficients are very accurate while they are obtained in much shorter time than Monte Carlo results. We provided the formulas for the coefficients for the case when multiple variables of a simulation element can be assumed to be stochastic, with the ability to apply Gaussian and Beta distributions.

References

[1] D. Xiu, "Fast numerical methods for stochastic computation: A review", Commun. Comput. Phys., vol. 5, pp. 242-272, Feb. 2009.

[2] P. Górniak, W. Bandurski, "A new approach to polynomial chaos expansion for stochastic analysis of EM wave propagation in an UWB channel," Wireless days, 23-25 March 2016, Toulouse.

[3] T. Bella, J. Reis, "The Spectral Connection Matrix for Any Change of Basis within the Classical Real Orthogonal Polynomials", Mathematics 2015, 3, 382-397; doi:10.3390/math3020382.

[4] http://mathworld.wolfram.com/HermitePolynomial.html (G. Colomer, pers. comm.).