Calculations of Frequency Dependent Transmission Line Model for Coupled Exponential Line.

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Abstract— **The paper presents an application of a fast and effective method of modeling a nonuniform interconnect with frequency dependent parameters for multiconductor exponential transmission line. The model uses the S-parameters. The approach is based on the method of successive approximations. The approximation of the scattering parameters is calculated. For the exponential line in steady state only second order approximation is needed. The transient state for trapezoidal source is also presented. For that case the fourth order approximation is needed. Comparisons of the calculated results with the exact calculations are performed for exemplary exponential transmission line.**

Keywords—Interconnect, VLSI, Exponential Transmission Line, Scattering Parameters, SPICE

I. INTRODUCTION

Simulation and modeling of integrated circuits and/or printed boards are ongoing challenge. In this area there is still a need for modeling of transmission lines in the time-domain.

Nonuniform Transmission Lines have a wide applicability in RF and microwave circuits. Exponential lines are an important category of NUTLs and there can be find many methods and approaches to model them.

For the nonuniform line there are derived many numerical and analytical methods of analysis which are described in a very rich literature dating back to the fifties of the last century. These methods operate mainly in steady state in frequency domain. Recently, eg. [1,2,3,6,7] appeared papers in which are presented approaches to analysis both in harmonic steady and transient state. In [1] the author presents the approximate analytical solution using chain matrix parameters of NTLs to calculate the arbitrary lossy and disperssive NTLs. Additionally it is worth mentioning two other works. In the first paper [2], the author presents an approach based on dyadic Green's function and vector fitting of per-unit-length impedance and admittance of transmission line to obtain a Z matrix of transmission line as a two-port. The line impedance and admittance are the sums of rational functions of complex frequency s, which facilitates the transformation to the time-domain and modeling in SPICE. The biggest problem is the necessity to take into account a large number of terms in every entry of the mentioned Z matrix. In [3], the same author has extended the above approach to weakly nonuniform transmission lines. In that case the author used results obtained for uniform case and parametric macromodeling to obtain the approximate Z matrix of the line. In both papers, the presented approach has been extended to the case of a multiconductor line. Quite recently in [7,8] the authors present an iterative and adaptive multi-step perturbation technique for the analysis of arbitrary NUTLs. In [8] this technique was applied to the analysis of the exponential transmission lines and it is shown to be accurate, but huge number of iteration has been needed to obtain required exactness.

On the other hand in paper [4] was developed a method to convert of differential telegrapher's equations into integral equations and next to solve them using the method of successive approximation. In that approach, we obtain a first order approximation of the solution in a simple analytical form which is valid for low loss transmission lines. The drawback of that approach was not including the skin effect and dielectric dispersion.

Our previous researches [6,9] gave the approximate form (by means of the successive approximation) of the scattering parameters both in frequency and time domain for uniform and nonuniform transmission line. For this purpose, as in [2,3], we used the concept of rational approximation of per-unit-length parameters of the line in the frequency domain. In this paper we generalized our approach to the case of nonuniform multiconductor transmission lines. Our solution has been applied to the analysis of two coupled exponential transmission lines.

The paper is organized as follows. The next section presents the integral equations approach to the nonuniform transmission line. In the third section, we employ the method of successive approximation to calculate the scattering parameters of an exponential transmission line. In the fourth section we present the results for the voltage response both for steady and transient state. We conclude in the last section.

II. TELEGRAPHER'S EQUATIONS IN ITEGRAL FORM

The equations for a multiconductor nonuniform, dispersive transmission line are the following:

$$
\begin{aligned}
-\frac{d\mathbf{V}(\mathrm{s,y})}{dz} &= \boldsymbol{Z}(s)r(z)\,\boldsymbol{I}(\mathrm{s,z}),\\
-\frac{d\boldsymbol{I}\,(\mathrm{s,y})}{dz} &= \mathbf{Y}(\mathrm{s})g(z)\boldsymbol{V}(\mathrm{s,z}),
\end{aligned}
\tag{1}
$$

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where

$$\boldsymbol{Z}(s) = \boldsymbol{Z}_o(\mathrm{s}) + \boldsymbol{Z}_1(\mathrm{s}), \mathbf{Y}(\mathrm{s}) = \boldsymbol{Y}_o(\mathrm{s}) + \boldsymbol{Y}_1(\mathrm{s}),$$

$$\boldsymbol{Z}_o(\mathrm{s}) = \boldsymbol{R} + s\boldsymbol{L}, \qquad \boldsymbol{Y}_o(\mathrm{s}) = \boldsymbol{G} + s\boldsymbol{C}$$

d-length of the line

$r(z)$, $g(z)$- transmission line taper.

In our method we introduce current waves instead of voltage and current into the transmission line equations (1). It is done, similarly as in [4], by transformations:

$$\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix}(z,s) = \boldsymbol{S}(z)\begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{I}\end{bmatrix}(z,s),$$

$$\boldsymbol{S} = \frac{1}{2}\begin{bmatrix}\mathbf{I}_n\sqrt{Y_c} & -\mathbf{I}_n\sqrt{Z_c}\\ \mathbf{I}_n\sqrt{Y_c} & \mathbf{I}_n\sqrt{Z_c}\end{bmatrix} \tag{2}$$

$Y_c = \sqrt{\frac{g(z)}{r(z)}} = f_c(z)^{-1}$, $\mathbf{I_n}$-identity nxn matrix.

Further we limit our consideration to the case of the lossles n-wire nonuniform transmission line.

Using transformation (2) we can pass to (3):

$$-\frac{d}{dz}\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix} = \begin{bmatrix} -s\boldsymbol{\Lambda}\sqrt{rg} & \mathbf{I_n}\frac{df_c(z)}{dz}\frac{1}{2f_c(z)}\\ \mathbf{I_n}\frac{df_c(z)}{dz}\frac{1}{2f_c(z)} & s\boldsymbol{\Lambda}\sqrt{rg}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix}. \tag{3}$$

where $\boldsymbol{\Lambda} = \mathbf{X^{-1}LP^{-1}} = \mathbf{PCX},$ where matrices $\mathbf{X}$, $\mathbf{P}$ are introduced for diagonalization of equation (1) as in [4]. Similarly as in [4, 6] we pass to the integral form of equations (3) as follow:

$$\begin{aligned}\boldsymbol{I}_-(s,z) &= e^{-s\boldsymbol{\Lambda}\int_z^d \sqrt{rg}dx}\boldsymbol{I}_-(s,d)\\ &+ \mathbf{1}\int_z^d \frac{df_c(z)}{dz}\frac{1}{2f_c(z)} e^{-s\boldsymbol{\Lambda}\int_z^\xi \sqrt{rg}dx}\boldsymbol{I}_+(s,\xi)d\xi\\ \boldsymbol{I}_+(s,z) &= e^{-s\boldsymbol{\Lambda}\int_0^z \sqrt{rg}dx}\boldsymbol{I}_+(s,0)\\ &- \mathbf{1}\int_0^z \frac{df_c(z)}{dz}\frac{1}{2f_c(z)} e^{-s\boldsymbol{\Lambda}\int_\xi^z \sqrt{rg}dx}\boldsymbol{I}_-(s,\xi)d\xi\end{aligned} \tag{4}$$

III. Scattering Parameters For Nonuniform Muticonductor Transmission Line

A. Scattering parameters for muticonductor-nonuniform transmission line

The current wave formulation allows us to obtain the scattering parameters. From the integral equations (4) similarly as in [4,6] we calculate the scattering parameters using the method of successive approximations or iterations. Let us consider the first order approximation. Then we obtain following formulas for the scattering parameters:

$$\begin{aligned}\boldsymbol{S}_{11}^{1}(s) &= \mathbf{1}\int_0^d \frac{df_c(z)}{dz}\frac{1}{2f_c(z)} e^{-s\boldsymbol{\Lambda}\int_0^\xi 2\sqrt{rg}dx}d\xi\\ \boldsymbol{S}_{12}^{1}(s) &= \boldsymbol{S}_{21}^{1}(s) = e^{-s\boldsymbol{\Lambda}\int_0^d \sqrt{rg}dx}\\ \boldsymbol{S}_{22}^{1}(s) &= -\mathbf{1}\int_0^d \frac{df_c(z)}{dz}\frac{1}{2f_c(z)} e^{-s\boldsymbol{\Lambda}\int_\xi^d 2\sqrt{rg}dx}d\xi\end{aligned} \tag{5}$$

The similar way we can calculate further order approximations [6]. Presented formulas for the scattering parameters (5) are for the decoupled transmission lines and can be easily transformed to the original lines by the relationships:

$$\begin{aligned}\boldsymbol{S}_{11}^{o}(s) &= \boldsymbol{P}^{-1}\boldsymbol{S}_{11}\boldsymbol{P},\\ \boldsymbol{S}_{12}^{o}(s) &= \boldsymbol{P}^{-1}\boldsymbol{S}_{12}\boldsymbol{P},\\ \boldsymbol{S}_{22}^{o}(s) &= \boldsymbol{P}^{-1}\boldsymbol{S}_{22}\boldsymbol{P},\end{aligned} \tag{6}$$

B. Scattering parameters for the exponential transmission line

Assuming the exponential transmission line we obtain some simplification in formula (5):

$$r(z) = e^{\alpha z}, g(z) = e^{-\alpha z}, \alpha = {}^{q}/_{d} \tag{7}$$

By substituting the above relationships to equations (5) and performing integrations we obtain the closed form formulas for the first approximation:

$$\boldsymbol{S}_{11}^{1}(s) = \frac{\alpha e^{-\boldsymbol{\Lambda}ds}\sinh(\boldsymbol{\Lambda}ds)}{2s}\boldsymbol{\Lambda}^{-1} \tag{8a}$$

$$\boldsymbol{S}_{22}^{1}(s) = -\frac{\alpha e^{-\boldsymbol{\Lambda}ds}\sinh(\boldsymbol{\Lambda}ds)}{2s}\boldsymbol{\Lambda}^{-1} \tag{8b}$$

$$\boldsymbol{S}_{12}^{1}(\omega) = \boldsymbol{S}_{21}^{1}(\omega) = e^{-\boldsymbol{\Lambda}ds} \tag{8c}$$

For the second approximation we obtain:

$$\boldsymbol{S}_{11}^{2}(s) = \frac{\alpha e^{-\boldsymbol{\Lambda}ds}\sinh(\boldsymbol{\Lambda}ds)}{2s}\boldsymbol{\Lambda}^{-1} \tag{9a}$$

$$\boldsymbol{S}_{22}^{1}(s) = -\frac{\alpha e^{-\boldsymbol{\Lambda}ds}\sinh(\boldsymbol{\Lambda}ds)}{2s}\boldsymbol{\Lambda}^{-1} \tag{9b}$$

$$\begin{aligned}\boldsymbol{S}_{12}^{2}(\omega) &= \boldsymbol{S}_{21}^{2}(\omega) =\\ &= e^{-\boldsymbol{\Lambda}ds} - \frac{\alpha^2 e^{-\boldsymbol{\Lambda}ds}}{16s^2}(e^{-2\boldsymbol{\Lambda}ds} + 2\boldsymbol{\Lambda}ds - 1)\boldsymbol{\Lambda}^{-2}\end{aligned} \tag{9c}$$

We can calculate next successive approximation and obtain the closed form formulas. The calculations are simple, but for further approximation they are longer so we do not show them in this paper. Each iteration improve the formulas for S_{11} and S_{22} or S_{12} and S_{21} respectively. The second approximation gives an improvement in S_{12}, so if we want to have a next improvement in S_{12} we need two extra iterations.

For the exemplary transmission line, described in next section, for 1 GHz analysis, the second order approximation was good enough. For the transient response of our exponential line we derived the fourth order approximation for S parameters. The results are shown in Fig.1. The lines have identical parameters, so the S parameters are the same for both lines.

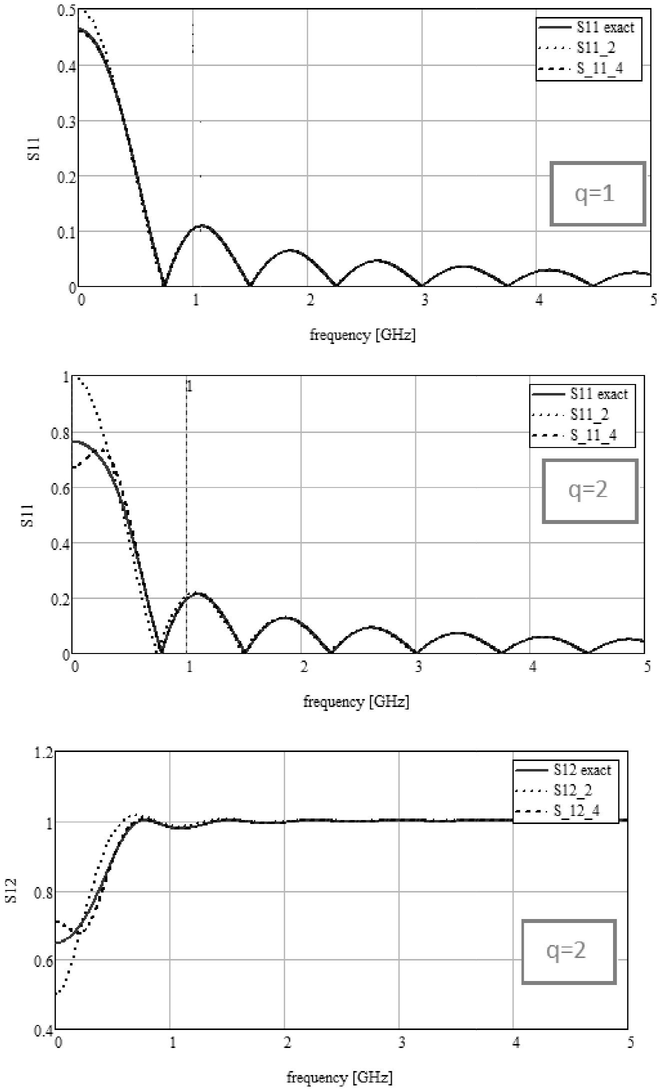

Fig.1 The S Parameters for the exponential transmission line of the second and fourth order approximation (for the both lines).

IV. RESULTS

As an example we have considered a nonuniform exponential coupled interconnect with frequency dependent parameters shown in Fig.2.

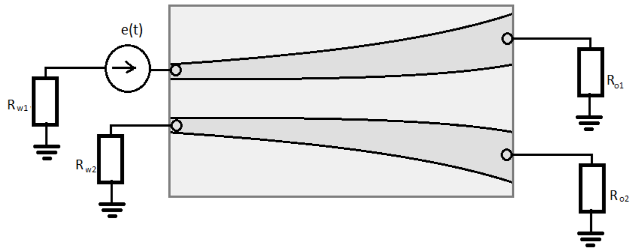

Fig.2 The considered system with exponential line.

We consider an example of the coupled two wire exponential line from [8]. The parameters of the line are:

$$\mathbf{L}(z) = \mathbf{L}_0 e^{\frac{qz}{l}}, \mathbf{C}(z) = \mathbf{C}_0 e^{-\frac{qz}{l}}$$

$$\mathbf{L}_0 = \begin{bmatrix} 171.4 & 18.65 \\ 18.65 & 171.4 \end{bmatrix} 10^{-9} H/m\,,$$

$$\mathbf{C}_0 = \begin{bmatrix} 65.7 & -7.15 \\ -7.15 & 65.7 \end{bmatrix} 10^{-12} F/m,$$

$$d = 0.2m, R_{w1} = R_{w2} = 50\Omega, R_{o1} = R_{o2} = 50e^{q}\Omega$$

On the base of scattering parameters one can calculate the formulas for the voltage response. The next figures (Fig 3-10) show the results for the considered exponential transmission line model.

For comparison we calculate scattering parameters of the exponential transmission line from the exact formula, what can be done in this case analytically.

First we considered the steady state for the sine source at 1GHz. Fig.3 and Fig.4 show the voltage response compared with exact one for q=2. We use the second order approximation to calculate our results. One can see that exactness of our second order approximation is very good.

Second example shows the circuit on Fig.2 but excited by a voltage pulse source (of the trapezoid shape A=2V, $T_r = T_f$ =500ps, T_{on}=2ns).

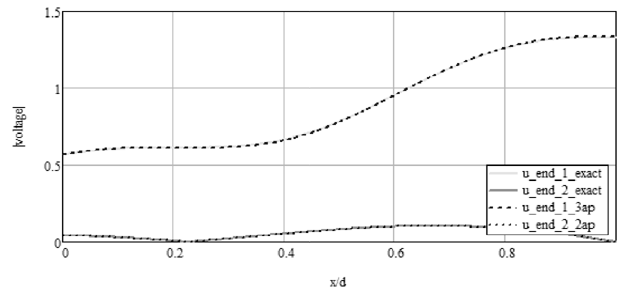

Fig.3 Voltage response, magnitude, q=2.

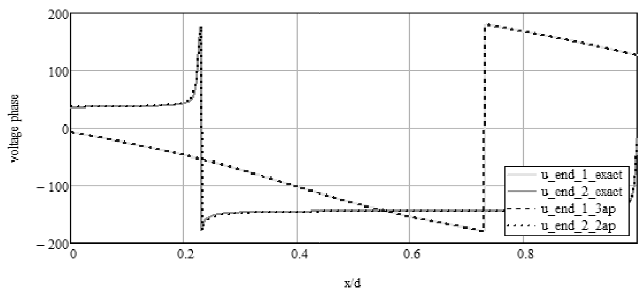

Fig.4 Voltage response, phase, q=2.

The near- and far-end voltages of the exponential transmission line (Fig.5-10) were obtained based on the fourth order approximation of scattering parameters. For comparison exact scattering parameters was calculated also. Next the voltages from frequency domain were subsequently transformed (IFFT) to the time domain.

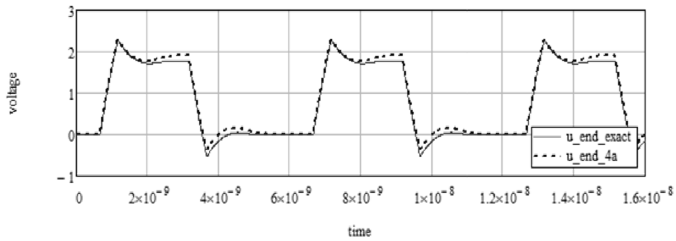

Fig.5 Far end voltages for trapezoidal source, excited line, q=2.

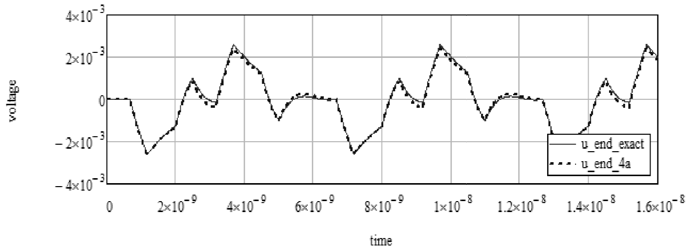

Fig.6 Far end voltages for trapezoidal source, crosstalk, q=2.

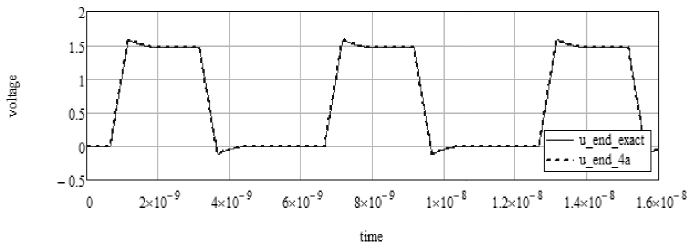

Fig.7 Far end voltages for trapezoidal source, excited line, q=1.

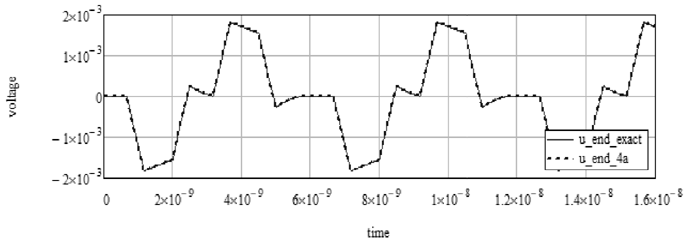

Fig.8 Far end voltages for trapezoidal source, crosstalk, q=1.

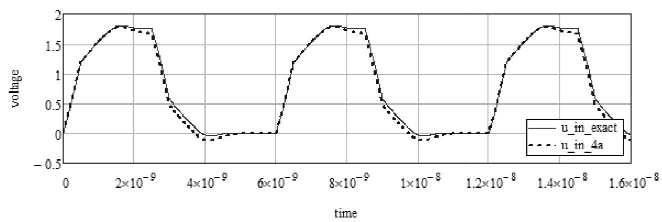

Fig.9 Near end voltages for trapezoidal source, excited line q=2.

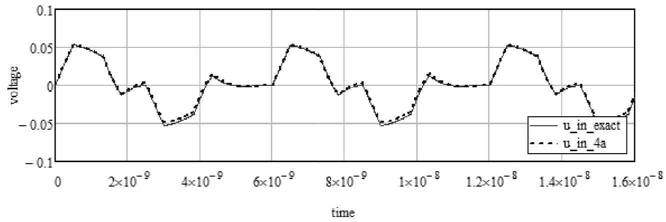

Fig.10 Near end voltages for trapezoidal source, crosstalk, q=2.

V. Conclusions

We have shown that it is possible to generalize the approach based on the method of successive approximation for the case of the multiconductor nonuniform transmission line with frequency dependent parameters. We can use the calculation for exponential transmission line and obtain approximate but closed form formulas for the S parameters. The approximation is satisfactory for the considered transmission line.

Compared with the approach based on dyadic Green's function and parametric macromodeling applied to weakly nonuniform transmission lines [2,3] the presented approach is simpler. The results obtained in [8] are similar but in our approach less approximations (iterations) are needed. If we are interested only in terminal voltages of the line the successive terms improving S parameters can be calculated numericaly for each frequency. Such a calculation are time consuming, but should be done once. Hence the presented approach permits the implementation of the model in the SPICE program.

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