Multiconductor Transmission Line Model with Frequency Dependent Parameters in Time Domain

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Abstract— In the paper we present fast and effective method of modeling of coupled interconnect in Spice simulator by means of S-parameters. The paper presents an approach based on the method of successive approximations, but taking into account the dependence on the frequency of line parameters. The concept is to use the rational approximation of the matrix of per-unit-length parameter of the line calculated for each frequency. In our approach we implemented the scattering parameters of transmission line to the Spice simulator.

Keywords — Interconnect, VLSI, Scattering Parameters, Transmission Line, Spice

I. INTRODUCTION

Modeling of transmission lines in the time-domain is an ongoing challenge for the people involved in the simulation of integrated circuits and/or printed circuit boards at high frequency. The literature on this subject is very rich and presenting it here is almost impossible. Among many methods and approaches we would like to focus on two, which include further references. In first paper [2] the author presents approach based on dyadic Green's function and vector fitting of per-unit-length impedances and admittances of transmission line to obtain Z matrix of n-port of multiconductor transmission line. The every entry of Z matrix is the sum of rational functions of complex frequency s, what facilitates the transformation to the time-domain and the modeling of circuit in SPICE. The biggest problem is the necessity to take account a large number of terms in every entry of mentioned Z matrix. On the other hand in paper [3] was developed method of conversion of differential telegrapher's equations into integral equations and next solving them by the method of successive approximation. In that approach we obtain first order approximation of the solution in simple analytical form, which is valid for low loss transmission lines. The drawback of that approach was not including the skin effect and dielectric dispersion.

In this paper we present an improved version of the approach based on the method of successive approximations [3], taking into account the line parameters dependence on the frequency. For this purpose, as in [3], we use the concept of rational approximation of the matrix of per-unit-length parameters of the line calculated for each frequency. In our approach we base on scattering parameters of n-wire transmission line. Such parameters for both frequency and time domains was obtained in [1]. In this paper implementation of the model into the Spice simulator is presented.

The paper is organized as follows. In next section there is presented the integral equations approach to the dispersive transmission line. In third section we apply the method of successive approximation to calculate the scattering parameters of multiconductor line and present the proposed model in SPICE. In fourth section we present an exemplary transmission line calculations. We conclude in the last section.

II. TELGRAHER'S EQUATIONS IN INTEGRAL FORM

A. *Telegrpher's equations for dispersive multiconductor transmission line*

Let us consider a multiconductor transmission line, consisting of N+1 conductors of which one is considered as reference one. The telegrapher's equations are following:

$$\begin{aligned} -\frac{d\mathbf{V}(\mathrm{p},\mathrm{y})}{dy} &= \left(\boldsymbol{Z_o}(\mathrm{p}) + \boldsymbol{Z_1}(\mathrm{p})\right)\boldsymbol{I}(\mathrm{p},\mathrm{z}) \\ -\frac{d\boldsymbol{I}(\mathrm{p},\mathrm{y})}{dy} &= \left(\boldsymbol{Y_o}(\mathrm{p}) + \boldsymbol{Y_1}(\mathrm{p})\right)\boldsymbol{V}(\mathrm{p},\mathrm{y}) \end{aligned} \tag{1}$$

where

$$\boldsymbol{Z}_o(\mathrm{p}) = \boldsymbol{R} + p\boldsymbol{L}, \qquad \boldsymbol{Y_o}(\mathrm{p}) = \boldsymbol{G} + p\boldsymbol{C}$$

$$\boldsymbol{Z}_1(\mathrm{p}) = \sum_{m=1}^{N_z} \frac{\boldsymbol{R}_m^z}{p + p_m^z}, \boldsymbol{Y_1}(\mathrm{p}) = \sum_{m=1}^{N_y} \frac{\boldsymbol{R}_m^y}{p + p_m^y}$$

$y = z/d\,,\ \tau = {}^{t}/_{T}\,,\ p = sT, T = d\sqrt{L_{11}^o\, C_{11}^o}$
d-length of the line
L_{11}^o - entry of original inductance matrix
C_{11}^o - entry of original capacitance matrix

In (1) matrices $\mathbf{Z}_1$ and $\mathbf{Y}_1$ are rational form of per-unit-length impedance and admittance of the multiconductor transmission line obtained as in [2] by means of vector fitting technique [5]. The next step is partial decoupling of the multiconductor transmission line. It is done, as e.g. in [3], by matrix transformations:

$$\boldsymbol{U} = \boldsymbol{XV}, \qquad \boldsymbol{J} = \boldsymbol{P}^{-1}\boldsymbol{I}$$

$$-\frac{d\boldsymbol{U}(\mathrm{p,y})}{dy} = (\boldsymbol{X}^{-1}\boldsymbol{RP}^{-1} + \boldsymbol{pX}^{-1}\boldsymbol{LP}^{-1} + \boldsymbol{X}^{-1}\boldsymbol{Z_1}(\mathrm{p})\boldsymbol{P}^{-1})\,\boldsymbol{J}(\mathrm{p,z}),$$
$$-\frac{d\boldsymbol{J}(\mathrm{p,y})}{dy} = (\boldsymbol{PGX} + \boldsymbol{pPCX} + \boldsymbol{PY_1}(\mathrm{p})\boldsymbol{X})\boldsymbol{V}(\mathrm{p,y}), \qquad (2)$$

where:

$$\boldsymbol{X} = \boldsymbol{L}^{\frac{1}{2}}\boldsymbol{W} diag\left[\frac{1}{\sqrt{\lambda_k}}\right], \quad \boldsymbol{P} = diag\left[1/\sqrt{\lambda_k}\right]\boldsymbol{W}^{-1}\boldsymbol{L}^{1/2}$$

$\boldsymbol{W}, \lambda_k$ -eigenvector and eigenvalues of matrix $\boldsymbol{L}^{\frac{1}{2}}\boldsymbol{C}\boldsymbol{L}^{\frac{1}{2}}$. Equations (2) is partially decoupled, matrices $\boldsymbol{X}^{-1}\boldsymbol{LP}^{-1} = \boldsymbol{PCX}$ are diagonal. Now we introduce current waves by matrix transformation:

$$\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix} = \frac{1}{2}\begin{bmatrix}\mathbf{1} & -\mathbf{1}\\ \mathbf{1} & \mathbf{1}\end{bmatrix}\begin{bmatrix}\boldsymbol{U}\\ \boldsymbol{J}\end{bmatrix} = \boldsymbol{S}\begin{bmatrix}\boldsymbol{U}\\ \boldsymbol{J}\end{bmatrix}.$$

After some matrix manipulation we obtain new form of telegrapher equations:

$$\frac{d}{dy}\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix} + p\begin{bmatrix}-\boldsymbol{\Lambda} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Lambda}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix} = \begin{bmatrix}\mathbf{P_1} & -\mathbf{P_2}\\ \mathbf{P_2} & -\mathbf{P_1}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix} + \begin{bmatrix}\mathbf{Q_1} & -\mathbf{Q_2}\\ \mathbf{Q_2} & -\mathbf{Q_1}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_-\\ \boldsymbol{I}_+\end{bmatrix}, \qquad (3)$$

where

$$\mathbf{P_1} = \frac{1}{2}\boldsymbol{X}^{-1}(\boldsymbol{X}^{-1}\boldsymbol{RP}^{-1} + \boldsymbol{PGX}),$$
$$\mathbf{P_2} = \frac{1}{2}\boldsymbol{X}^{-1}(\boldsymbol{X}^{-1}\boldsymbol{RP}^{-1} - \boldsymbol{PGX}),$$
$$\mathbf{Q_1} = \frac{1}{2}\boldsymbol{X}^{-1}(\boldsymbol{X}^{-1}\boldsymbol{X}^{-1}\boldsymbol{Z_1}(\mathrm{p})\boldsymbol{P}^{-1} + \boldsymbol{PY_1}(\mathrm{p})\boldsymbol{X}),$$
$$\mathbf{Q_2} = \frac{1}{2}\boldsymbol{X}^{-1}(\boldsymbol{X}^{-1}\boldsymbol{X}^{-1}\boldsymbol{Z_1}(\mathrm{p})\boldsymbol{P}^{-1} - \boldsymbol{PY_1}(\mathrm{p})\boldsymbol{X}).$$

In (3) the diagonal entries of matrix $\mathbf{P_1}$ we move to the left side and after some manipulations we obtain (4).

$$\frac{d}{dy}\left[exp\left[-(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda})\right]\boldsymbol{I}_-\right] = exp\left[-(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda})\right]\mathbf{P}'_{\mathbf{10}}\boldsymbol{I}_- - exp\left[-(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda})\right]\mathbf{P}'_{\mathbf{2}}\boldsymbol{I}_+,$$
$$\frac{d}{dy}\left[exp\left[(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda})\right]\boldsymbol{I}_+\right] = exp\left[(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda})\right]\mathbf{P}'_{\mathbf{2}}\boldsymbol{I}_- - exp\left[(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda})\right]\mathbf{P}'_{\mathbf{10}}\boldsymbol{I}_+, \qquad (4)$$

where

$$\mathbf{P}'_{\mathbf{10}} = \mathbf{P_{10}} + \mathbf{Q_1}, \qquad \mathbf{P}'_{\mathbf{2}} = \mathbf{P_2} + \mathbf{Q_2}.$$

B. *Integral equations for dispersive multiconductor transmission line*

Integrating first of equations (4) from y to 1 and the second from 0 to y we obtain:

$$\boldsymbol{I}_-(p,y) = exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda}\right)(1-y)\right]\boldsymbol{I}_-(p,1) - \int_y^1 exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda}\right)(\xi-y)\right]\mathbf{P}'_{\mathbf{10}}\boldsymbol{I}_-(p,\xi)d\xi + \int_y^1 exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda}\right)(\xi-y)\right]\mathbf{P}'_{\mathbf{2}}\boldsymbol{I}_+(p,\xi)d\xi, \qquad (5a)$$

$$\boldsymbol{I}_+(p,y) = exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda}\right)y\right]\boldsymbol{I}_+(p,0) + \int_0^y exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda}\right)(y-\xi)\right]\mathbf{P}'_{\mathbf{10}}\boldsymbol{I}_-(p,\boldsymbol{\xi})d\boldsymbol{\xi} - \int_0^y exp\left[-\left(\mathbf{P_{1_{k,k}}} + p\boldsymbol{\Lambda}\right)(y-\xi)\right]\mathbf{P}'_{\mathbf{2}}\boldsymbol{I}_+(p,\boldsymbol{\xi})d\boldsymbol{\xi}\mathrm{q} \qquad (5a)$$

Equations (5) are integral telegrapher's eqs. and can be solved analytically or numerically. We calculate now (5) by means of the method of successive approximations.

III. Scattering Parameters of the Multiconductor Transmission Line and the SPICE Model

The first order approximation is not difficult to obtain (see[2]). Here we are giving first order approximation of the first of equations (5) and it has the following form:

$$\boldsymbol{I}_-(p,y) = e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(1-y)}\boldsymbol{I}_-(p,1) - \int_y^1 e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(\xi-y)}\,\mathbf{P}'_{\mathbf{10}}e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(1-\xi)}d\xi\,\boldsymbol{I}_-(p,1) + \int_y^1 e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(\xi-y)}\,\mathbf{P}'_{\mathbf{2}}\,e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)\xi}d\xi\,\boldsymbol{I}_+(p,0), \qquad (6)$$

Substituting y=0 in (6) we obtain the relationships:

$$\boldsymbol{I}_-(p,0) = \int_0^1 e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(\xi-0)}\,\mathbf{P}'_{\mathbf{2}}\,e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)\xi}d\xi\,\boldsymbol{I}_+(p,0) + \left[e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(1-y)} - \int_0^1 e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(\xi-0)}\,\mathbf{P}'_{\mathbf{10}}e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(1-\xi)}d\xi\right]\boldsymbol{I}_-(p,1). \qquad (7)$$

In (7) we can easily identify scattering parameters as:

$$\boldsymbol{S}_1(p) = \int_0^1 e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(\xi-0)}\,\mathbf{P}'_{\mathbf{2}}\,e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)\xi}d\xi$$
$$\boldsymbol{S}_2(p) = e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(1-y)} - \int_0^1 e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)\xi}\,\mathbf{P}'_{10}e^{-\left(\mathbf{P_{1_{k,k}}}+p\boldsymbol{\Lambda}\right)(1-\xi)}d\xi. \qquad (8)$$

A. *Scattering matrices in frequency domain*

Calculation of integrals (8) is straightforward and the results are following

$$S_1(p)_{m,i,j} = \left[P_{2_{i,j}} + Q_2(p)_{m,i,j}\right]\cdot \frac{1-\exp\left(-\left(P_{1_{i,i}}+P_{1_{j,j}}+p(\lambda_i+\lambda_j)\right)\right)}{P_{1_{i,i}}+P_{1_{j,j}}+p(\lambda_i+\lambda_j)}, \qquad (9a)$$

$$S_2(p)_{m,i,j} = e^{-\left(P_{1_{i,i}}+p\lambda_i\right)} + \left(P_{10_{i,j}} + Q_1(p)_{m,i,j}\right)\frac{e^{-\left(P_{1_{i,i}}+p\lambda_i\right)} + e^{-\left(P_{1_{j,j}}+p\lambda_j\right)}}{P_{1_{i,i}} - P_{1_{j,j}} + p(\lambda_i - \lambda_j)}, \qquad (9b)$$

where

$$Q_{1/2}(p)_{m,i,j} = \frac{1}{2}\left[\frac{Z_{2_{m,i,j}}}{p+p_m^z} +/- \frac{Y_{2_{m,i,j}}}{p+p_m^z}\right],$$
$$Z_{2_{m,i,j}} = [\boldsymbol{X}^{-1}\boldsymbol{R}_m^z\boldsymbol{P}^{-1}]_{i,j},\; Y_{2_{m,i,j}} = [\boldsymbol{PR}_m^z\boldsymbol{X}]_{i,j},$$

B. *Spice model*

Our proposed model of n-wire transmission line (Fig.1) uses the scattering parameters determined in the previous sections. In the first step we made a partial decoupling of the n-wire transmission lines with the use of matrix X and P, as it was done in the first section. This procedure is illustrated in Fig.2.

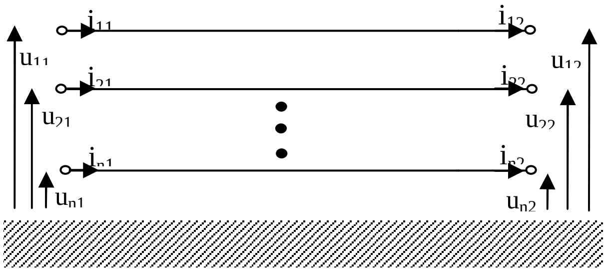

Fig.1 Multiconductor (n-wire) transmission line.

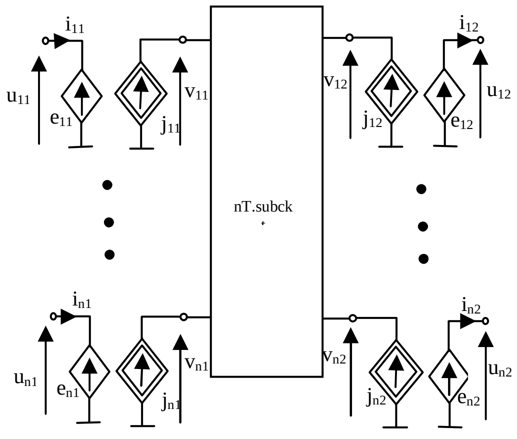

Fig.2 Spice model of multiconductor transmission line, where $e_{k,1/2} = \sum_{m=1}^{n} x_{k,m} v_{m,1/2}$, $j_{k,1/2} = \sum_{m=1}^{n} p_{k,m} i_{m,1/2}$.

Then, basing on equations (10,9), we create subcircuit (nT.subckt) containing partially decoupled transmission lines model using scattering parameters. Fig.3 shows an equivalent circuit of the k-th transmission line (k = 1, ..., n)

$$\boldsymbol{I}_-(p,0) = \boldsymbol{S_1}(p)\boldsymbol{I}_+(p,0) + \boldsymbol{S_2}(p)\boldsymbol{I}_-(p,1), \qquad (10a)$$

$$\boldsymbol{I}_+(p,1) = \boldsymbol{S_2}(p)\boldsymbol{I}_+(p,0) + \boldsymbol{S_1}(p)\boldsymbol{I}_-(p,1) \qquad (10b)$$

where

$$\boldsymbol{I}_-(p,0) = \boldsymbol{b_1}(p), \ \boldsymbol{I}_+(p,1) = \boldsymbol{b_2}(p),$$

$$\boldsymbol{I}_+(p,0) = \boldsymbol{a_1}(p), \qquad \boldsymbol{I}_-(p,1) = \boldsymbol{a_2}(p).$$

IV. Results

As an example we have considered one-wire transmission line with frequency dependent parameters. The dependence of the per-unit-length parameters Z_o, Z_1, Y_o and Y_1 on frequency can be calculated by means of electromagnetic field solvers e.g. program LINPAR [4]. However, in our

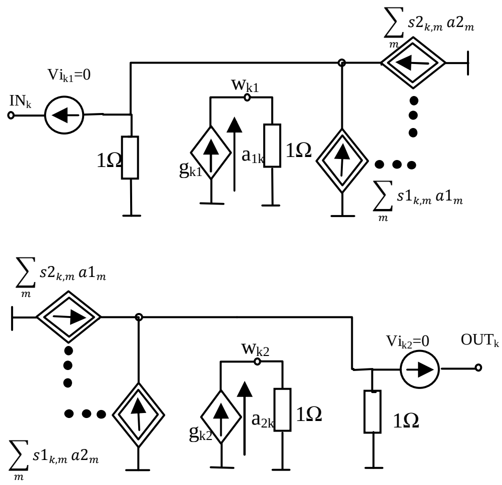

Fig.3 Subcircuit nT.subckt (Fig.2), k,m = 1,…,n.

example, we used the following relationships to demonstrate the skin effect (Fig.4) and dielectric dynamics:

$$Z_L(p) = R_0 + pL_0 + \left(0.1 + 10^{-4.5}\sqrt{p}\,\right) = Z_0(p) + Z_1(p),$$

$$Y(p) = G_0 + pC_0\left(1 + \frac{\varepsilon_s/\varepsilon_\infty - 1}{1 + p\tau}\right) = Y_0(p) + Y_1(p),$$

where
$R_0=10\Omega$, $L_0= 2nH$, $G_0 =10\mu S$, $\varepsilon_s = 4$, $\varepsilon_\infty =1$, $\tau = 2ns$.

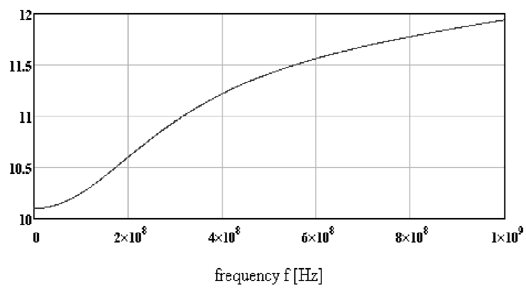

Fig.4 A typical frequency dependence of the real part of longitudinal impedance (resistance) of the line.

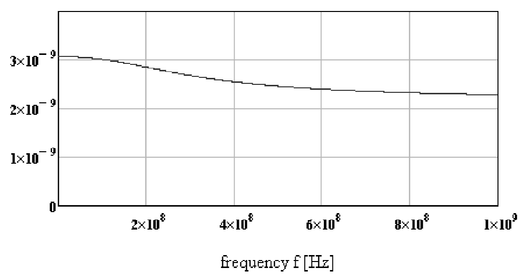

Fig.5 Transmission line inductance dependence on frequency.

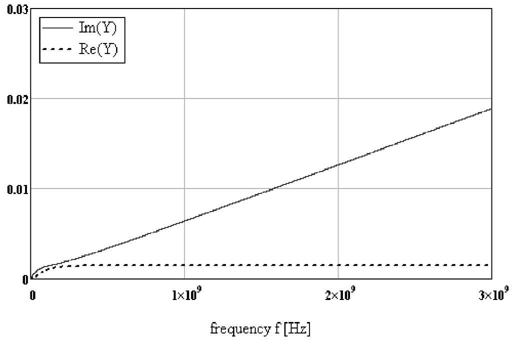

Fig.6 Dependence of shunt conductance and shunt reactance of the line on frequency.

The above relationships on the per-unit-length parameters as a functions of frequency were approximated by means of rational functions using very efficient algorithm-vector fitting [5,6]. Than we have implemented the proposed transmission line model into the Spice simulator. Simulated circuit consists of the voltage pulse sources (of the trapezoid shape A = 2V, T_r= T_f = 500ps, T_{on} = 2ns) with source resistance R_s=150Ω and transmission line loaded by capacitor C_L=1pF.

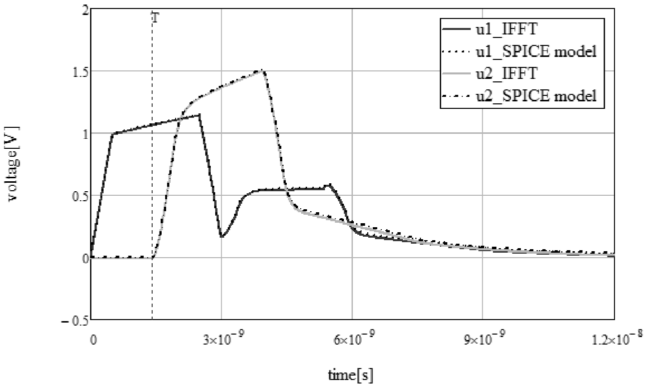

Fig.7 Comparison the SPICE model with IFFT calculation of the same model for the considered transmission line with frequency dependent p-u-l parameters.

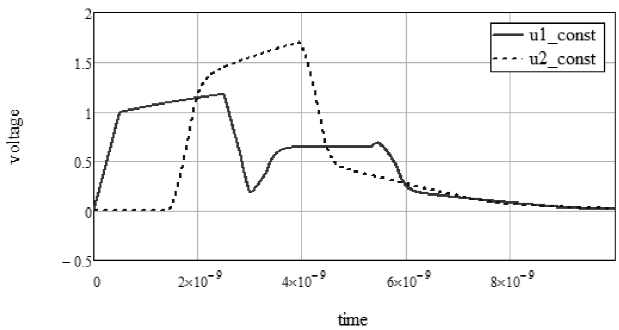

Fig.8 Input u1_const and output u2_const voltages at the near and far ends of the considered transmission line with constant p-u-l parameters.

The first for constant parameters (Y_0 and Z_0), and the second one for variable parameters as a function of the frequency discussed above and shown in Figs 4, 5 and 6 (Y_0+Y_1 and Z_0+Z_1). Voltage at the input and output lines for both sets of data are shown in Figs 7-9. In Fig.9 can be seen pulse distortion caused by the skin effect and dielectric permittivity dependence on frequency. The output voltage has lower magnitude. The comparison of the model in SPICE with IFFT calculation are presented in Fig 7. The IFFT is done to the same set of simplified formulas which are modeled in SPICE.

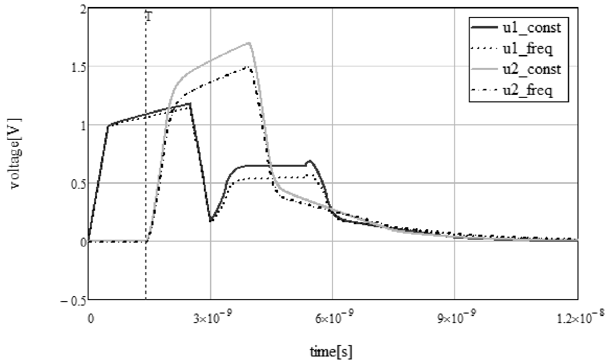

Fig.9 comparison of voltages at the near and far ends of the considered transmission line for constant and frequency dependent p-u-l parameters.

V. Conclusions

We have shown that it is possible to generalize approach based on the method of successive approximation for the case of multiconductor transmission line with frequency dependent parameters. As a result we obtain closed form (it means first order approximation) of scattering parameters of multiconductor transmission line (first order approximation) both in frequency and time domain. In the case of low loss transmission line an approximation gives very good results. Comparing with the approach based on dyadic Green's function [2] the presented approach is simpler, of course assuming sufficiently small losses of the multiconductor transmission line. The presented approach permits for implementation of the model to the SPICE. Currently we are working on the implementation of the model of n-wire transmission line into the program SPICE.

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