New Analytical PCE Coefficients for Uncertainty Quantification in Ray-Tracing Modeling

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***Abstract*— Ray-tracing simulations of stochastic electromagnetic fields are considered in the paper. The author uses polynomial chaos expansion (PCE) coefficients for uncertainty quantification. The author introduces the new effective method, in terms of accuracy and calculation speed, for analytical derivation of PCE coefficients in ray-tracing modelling. The analytical PCE coefficients can be recalculated very fast when it is necessary to change probability densities of random variables of a simulation. A ray-tracing simulation of exemplary indoor scenario is used to compare the new method with general polynomial chaos (gPC) approach which requires numerical calculation of PCE coefficients for each set of probability densities of simulation random variables.**

Keywords*— *simulation, stochastic electromagnetic fields, polynomial chaos, ray-tracing.

I. Introduction

The paper presents an method for fast calculation of coefficients of the general polynomial chaos (gPC) expansion [1] applicable to ray-tracing modeling and simulation of propagation of electromagnetic (EM) fields in a wireless channel with uncertainties of its physical parameters. General polynomial chaos has become a widely adopted technique in the area of analysis of stochastic EM fields [2-3]. The main elements of the gPC theory are polynomial chaos expansion (PCE) coefficients which are used to find stochastic moments and to perform sensitivity analysis of a considered function $Y(\xi)$ of stochastic variables $\xi = \{\xi_0, \xi_1, \xi_2,...\}$. Then the calculated stochastic moments can be used to find the corresponding probability density function (PDF) and cumulative distribution function (CDF) of $Y(\xi)$. In the paper the author proposes a very effective, accurate and fast, method for calculation of PCE coefficients in ray-tracing modelling. Nowadays, to the best knowledge of the author, the PCE coefficients need to be calculated numerically using Galerkin projection, collocation and optimization methods. These numerical techniques are implemented in an open access MATLAB-based package called UQLab [4] which is used by the author to obtain reference results in analysis of effectiveness of the proposed method for ray-tracing stochastic modeling. The algorithms that are used nowadays to derive the PCE coefficients of a considered $Y(\xi)$ require to perform substantial number of evaluations of $Y(\xi)$ what for ray-tracing modeling means substantial number full simulations of a scenario under consideration [5]. Furthermore these simulations and consequent numerical calculations needs to be performed for each set of random variables ξ for which one decide to test the considered simulation scenario. The aim of the author is to omit these inconveniences. The author provides the method for fast analytical calculation of the PCE coefficients for ray-tracing modeling during one full simulation of a scenario under consideration.

The paper is organised as follows. In Section II the new method for fast calculation of PCE coefficients for ray-tracing modelling is presented. The exemplary analysis of effectiveness of the proposed method is presented in Section III. The summary of the paper is given in Section IV.

II. The new PCE coefficients for ray-tracing modeling

The main point of gPC theory is to expand function of random variables $Y(\xi)$ using orthogonal basis of polynomials for a given joint probability density $p(\xi)$ as follows.

$$Y(\xi) = \sum_q c_q \cdot \psi_q(\xi) \tag{1}$$

where q is the number of row in a multi-index matrix defined according to the Askey scheme [1], $\psi_q(\xi)$ is a multivariate polynomial which is orthogonal with respect to orthogonal basis associated with a given joint probability density [1], while c_q are the PCE coefficients which can be calculated using Galerkin projection as follows [1]:

$$c_q = \frac{1}{\gamma_q}\langle Y(\xi), \psi_q(\xi)\rangle = \frac{1}{\gamma_q}\int_\Omega Y(\xi)\cdot\psi_q(\xi)\cdot p(\xi)d\xi \tag{2}$$

where Ω is the support of joint probability density $p(\xi)$, while γ_q is a normalisation factor [1]:

$$\gamma_q = \langle \psi_q(\xi), \psi_q(\xi)\rangle \tag{3}$$

For multivariate case $\psi_q(\xi)$ is the product of Hermite polynomials for a Gauss PDF, Jacobi polynomials for a Beta PDF, etc. [1]. After the PCE coefficients of $Y(\xi)$ are found, the stochastic moments of $Y(\xi)$ can be calculated as in [1].

It can be shown [6, 7], that (2) can be solved analytically if $Y(\xi)$ is first approximated for support Ω_0, which comprises Ω. In ray-tracing simulation random variables ξ can model, e.g., random permittivity and conductivity of walls for an indoor scenario. In order to find the approximation of $Y(\xi)$ for support Ω_0 it is enough to approximate reflection and diffraction coefficients for support Ω_0 which can be defined according to the data given in literature. It can be shown, that when these coefficients are approximated during pre-processing, the PCE expansion of $Y(\xi)$ can be found during one simulation for support Ω of any $p(\xi)$ which is a product of Gauss and Beta PDFs. This method is much more efficient than a collocation method for which numerous simulation repetitions are necessary. The general form of PCE expansion of $Y(\xi)$ for the case of the proposed method can be written as follows:

$$Y(\xi) \approx \sum_r \sum_k c_k^{(r)} \psi_k^{(r)}\left(\xi^{(r)}\right) \tag{4}$$

where r indicates the ray group which interacts with walls or edges represented by the same set of random independent permitivities and conductivities modeled by $\xi^{(r)}$, while k is the index of a polynomial, as well as PCE coefficient in the ray group with number r.

Taking advantage of (4) a mean and a standard deviation of $Y(\xi)$ can be written as:

$$\mu_Y = \sum_r c_0^{(r)} \tag{5}$$

$$\sigma_Y = \sqrt{\sum_r \sum_{k>0} \gamma_k^{(r)} \cdot c_k^{(r)2}} \tag{6}$$

The function of the ray group can be written as follows:

$$y^{(r)}\left(\xi^{(r)}\right) \approx \sum_k c_k^{(r)} \psi_k^{(r)}\left(\xi^{(r)}\right) \tag{7}$$

Using a ray tracing principles the function of the ray group can be also expressed in terms of coefficients of wave phenomena (reflection, refraction, diffraction) as follows:

$$y^{(r)}\left(\xi^{(r)}\right) = \sum_i e^{-j \cdot \beta_0 \cdot s_i} \cdot \prod_m FWP\left(\boldsymbol{\theta WP}^{(m)}, \xi^{(m)}\right) \tag{8}$$

where i is the entry number of a single ray in the ray group r, β_0 is a phase constant in the air, s_i is a length of a path of the i-th ray in the group of rays, the function of wave phenomena $FWP(\boldsymbol{\theta WP}^{(m)},\xi^{(m)})$ represents single or multiple reflection or diffraction coefficient corresponding to the i-th ray in the ray group with number r (e.g. multiple reflection coefficient is a product of a single reflection coefficients $\Gamma(\theta,\sigma,\varepsilon_r)$ associated with the same surface). The function of wave phenomena depends on a deterministic vector of reflection or diffraction angles $\boldsymbol{\theta WP}^{(m)}$ and a random vector $\xi^{(m)}$ which is a subset of $\xi^{(r)}$. Vector $\xi^{(m)}$ models random permittivity, conductivity, etc., associated with a single or multiple reflection or diffraction coefficient. It is important to notice that random variables $\xi^{(m)}$ corresponding to any pair of different m values do not overlap. When all the vectors $\xi^{(m)}$ are gathered together they compose $\xi^{(r)}$ for each i-th ray.

In order to find the PCE coefficients in (7) the author derives analytical formulas for PCE coefficients of $FWP(\xi^{(m)})$. The single reflection, diffraction coefficients corresponding to dielectric surfaces and edges are given in literature and are adopted for ray-tracing modeling. The ranges of variability (domain Ω_0) of physical parameters for these coefficients can be a-priori established following e.g. measurement results given in literature.

The PCE expansion of $FWP(\xi^{(m)})$ for domain Ω which is the subset of Ω_0 can be written as follows:

$$FWP\left(\xi^{(m)}\right) \approx \sum_{n=0}^{N} c_n^{(m)}\left(\boldsymbol{\theta WP}^{(m)}\right)\psi_n^{(m)}\left(\xi^{(m)}\right) \tag{9}$$

where n is the index of a polynomial that is used for the orthogonal expansion of the function of wave phenomena. It must be noted that orthogonal polynomials in (4) and (7) are constructed (e.g. in the process of multiplying) using the polynomials in (9). The PCE coefficients in (9) are the functions of Ω and parameters of joint PDF of $\xi^{(m)}$. In order to find the analytical form of $c_n^{(m)}(\boldsymbol{\theta WP}^{(m)})$, the results from [6, 7] can be applied. In a result they can be formulated as follows.

$$c_n^{(m)} \approx \frac{1}{\gamma_n^{(m)}} \sum_{p=0}^{P} d_p^{(m)} S_{p,n}^{(m)}\left(\xi^{(m)}\right) \tag{10}$$

where p indicates the index of polynomial that is used to obtain the orthogonal expansion of $FWP(\boldsymbol{\theta WP}^{(m)},\xi^{(m)})$ in domain Ω_0 (Ω is a subset of Ω_0). Constants $d_p^{(m)}$ are obtained for the case when each polynomial of orthogonal expansion is a product of Hermite polynomials as in [6, 7]. They are tabulated, because the expansion of wave phenomenon coefficient for domain Ω_0 is performed only once. Function $S_{p,n}^{(m)}(\xi^{(m)})$ is the product of corresponding functions that are presented in [6, 7] for the case of univariate expansion. The components of this product for the case of a Gauss PDF can be written as follows:

$$S_{p,n}^{G}(\xi) = \frac{\left(h^{Gauss}\right)^p}{\left(\sqrt{1-\left(g^{Gauss}\right)^{-2}}\right)^p} \sum_{j=0}^{p} \left(\sqrt{\frac{\left(g^{Gauss}\right)^2 - 1}{\left(h^{Gauss}\right)^2}}\right)^j Q(j,p,n) \tag{11}$$

and for Beta PDF in the following form:

$$S_{p,n}^{B}(\xi) = \sum_{j=0}^{p} \binom{p}{j} H_j\left(h^{Beta}\right)\left(g^{Beta}\right)^{p-j} I_n \tag{12}$$

Interpretation and calculation of expressions g^{Gauss}, h^{Gauss} and $Q(j,p,n)$ can be made according to formulas (10), (11) and (18), respectively, which are given in [6]. The corresponding formulas for g^{Beta}, h^{Beta} and I_n are given in [7] by (12), (13) and (19) – (20), respectively.

III. Simulation Example

In the previous Section the new method for calculation of PCE coefficients of random EM fields calculated using ray-tracing technique was presented. The proposed form of PCE expansion is given by (4) and revealed by formulas (7) – (12). The calculation of the these PCE coefficient is purely analytical. It means that numerical integration as in (2) is not required to calculate these PCE coefficients. The presented method is verified in this Section using a simple simulation scenario shown in Fig. 1.

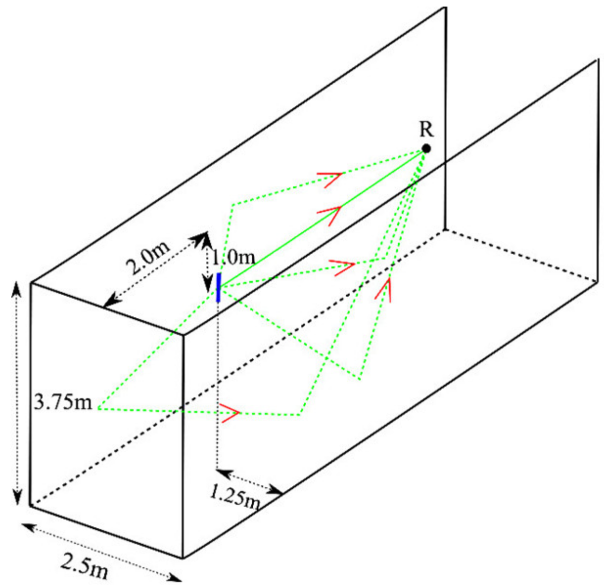

Fig. 1. The considered scenario for ray-tracing simulation.

We can see in Fig. 1 a corridor of height 3.75m and width 2.5m. The dipole antenna is placed 1m below the ceiling. The radiated electromagnetic wave is a 6GHz carrier. The radiated power is 0 dBm EIRP. The wave is vertically polarised. We can see some rays leading from transmitting antenna to an exemplary observation point ***R*** in Fig. 1 (up to 6 reflections for a single ray are considered in a full simulation). The direct ray leads along the side walls. The attenuation of the EM wave is analysed along this direct ray within 2m - 5m distance from the transmitting antenna. Each of the constitutive parameters of walls, ceiling and floor is a uniform random variable whose support has lower limit *a* and higher limit *b*. These limits are given in Table I.

TABLE I. MATERIAL PARAMETERS OF SURFACES INCLUDED IN FIG. 1.

Surface	σ [s/m]		ε_r	
	a	b	a	b
Walls	1	2	4	6
Floor	1	3	7	9
Ceiling	1	3	6	8

The results of simulations are shown in Figs 2-3, where the new method results (dotted blue graphs) are compared against the multiple simulations approach [5] (solid red lines). The results are given in a dB scale. The simulations were implemented in MATLAB environment using UQLab [4].

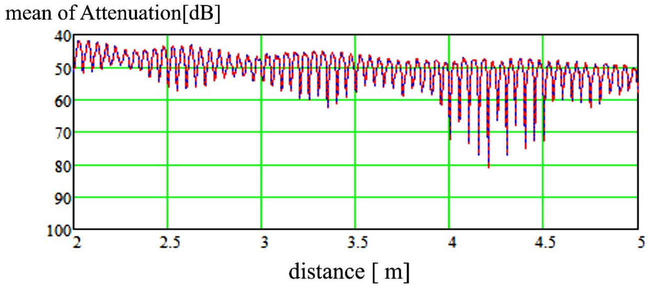

Fig. 2. The mean of attenuation of an electric field along the direct ray in Fig. 1 calculated with the new method – dotted blue graphs and with multiple simulations [5] – solid red line.

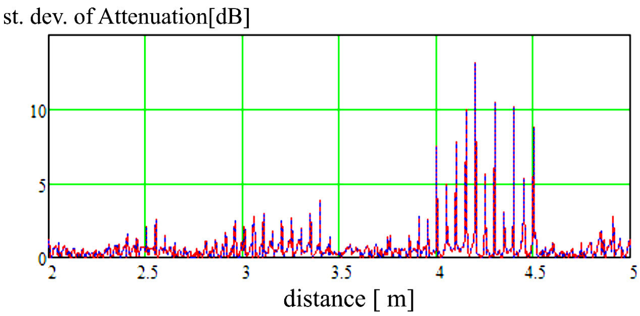

Fig. 3. The standard deviation of attenuation of an electric field along the direct ray in Fig. calculated with the new method – dotted blue graphs and with multiple simulations [5] – solid red line.

The exemplary simulation results shown in Figs 2-3 indicate a great agreement of the new method results (one simulation) with the reference results obtained with multiple simulations using UQLab numerical calculations. The speedup of the proposed method is shown in Table II. The speedup is defined as the result of a division of the time associated with multiple simulation approach by the time associated with the new proposed method.

TABLE II. THE SPEEDUP PROVIDED BY THE METHOD PRESENTED IN SECION II.

Method	Time [min]	Speedup
Multiple simulations	35.54	1
New method	1.56	22.78

IV. CONCLUSIONS

The author presented in the paper the new effective, in terms of accuracy and calculation time, method for calculation of PCE coefficients in ray-tracing modeling. The proposed method provides a great speedup of simulations of stochastic EM fields what was shown in the previous Section. The new form of PCE coefficients are given in a closed form, and are the analytical functions of parameters of PDFs of random variables ξ (e.g. μ and σ for a Gauss PDF or α and β for a Beta PDF). Therefore the new PCE coefficients can be quickly recalculated when there is a need to test the considered simulation scenario for many different sets of PDFs of random variables ξ.

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