Sensitivity of threshold crossing time to geometrical dimensions of VLSI interconnects

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Abstract—**The paper presents a sensitivity analysis of threshold crossing time for the step and ramp response of VLSI interconnects to geometrical dimensions of microstrip model of higher level interconnects. The method is based on threshold crossing time obtained for analytical form of VLSI output response calculated using multiple scales method.**

Index Terms—**sensitivity; threshold crossing time; VLSI interconnects.**

I. INTRODUCTION

In modern VLSI technology with the device scaling and number of devices growth the problem of interconnects becomes more significant. Especially the higher level global interconnects generates delays larger then devices delay. Because of high inductance of such interconnects [3] and the resistance which cannot be neglected during the modeling RLC transmission line must be considered. The problem of RLC transmission line excited by voltage source and loaded with capacitance, what are typical model for gate input and output (Fig. 1) is not analytically solvable. Then there are proposed some method to calculate the output response and threshold crossing time [2] of the typical higher level interconnects. Calculating the sensitivity, which characterizes the influence of changing parameter value on system functionality, is a not easy task, when there is not possible to obtain the formula of the parameter we want to consider. Sensitivity analysis permits to estimate, which of system parameters affect more and which one of them less on it. Sensitivity permits for identification of the number of variables, which have critical influence on operation of the circuit. The problem of sensitivity calculation is considered e.g. in [8].

In the work we present the method of calculating the sensitivity of the threshold crossing time to geometrical parameters of the interconnect. First we calculate the analytical form of step and ramp response using the multiple scales method. In this work only the basic rules and the results are presented, the complete calculations are presented in [7]. Then the equation for calculating the threshold crossing time is constructed, and the threshold crossing time is obtained. From the previous equation there is a possibility to sensitivity of the threshold crossing time calculation.

In the next section we present the theoretical calculation of the output responses and the threshold crossing time, in the third section we show the way we calculate the sensitivity. We conclude in the last section.

II. THE THRESHOLD CROSSING TIME CALCULATION

A. General foundations

In our work we shortly present the method of computation the step and ramp responses for higher level wires assuming RLC transmission model and the global resistance less than lossless interconnect impedance.

$$\frac{R}{2Z_0} \leq 1 \tag{1}$$

That assumption allows treat the line as low-loss line with high inductance influence. The considered system is shown in Fig.1.

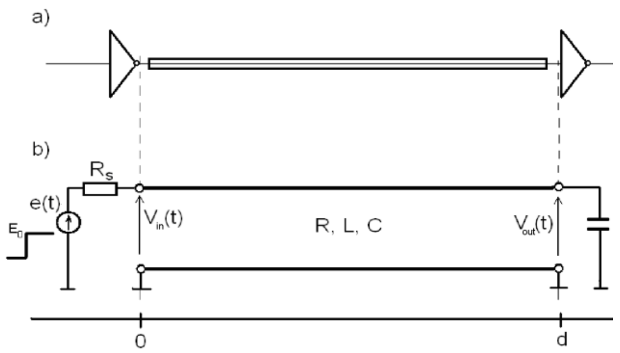

Fig. 1. a) The considered system inverter-interconnect-inverter, b) the model of the considered system

For the system of differential equation (2) with initial and boundary conditions (3,4) we could use the perturbation method of multiple scales, using the $\varepsilon = \frac{R_t}{Z_0}$ as perturbation parameter ($R_t = R \cdot d$).

$$\begin{cases} -\dfrac{\partial v}{\partial x} = Ri + L\dfrac{\partial i}{\partial t}, \\ -\dfrac{\partial i}{\partial x} = C\dfrac{\partial v}{\partial t} \end{cases} \tag{2}$$

$$i(x,0) = 0, \quad v(x,0) = 0, \tag{3}$$

$$e(t) - R_s i(0,t) = v(0,t),$$

$$-i(d,t) = C_0 \frac{\partial v(d,t)}{\partial t}, \tag{4}$$

Where R, L, C – line parameters, C0 – input inverter capacitance, Rs – output inverter resistance, i(x,t), v(x,t) – current and voltage in line, respectively, d – line length, t, x – time and space variable, respectively.

After scaling system (2) to obtain the perturbation parameter in the system, we can observe, that resistance of the interconnect influence only for perturbation parameter value. After scaling we have:

$$y = \frac{x}{d},\ \tau = \frac{t}{\sqrt{L_t C_t}},\ \tilde{v} = -\sqrt{\frac{C_t}{L_t}}v,\ \tilde{e} = \sqrt{\frac{C_t}{L_t}}e,\ \beta = \sqrt{\frac{C_t}{L_t}}R_s \tag{5}$$

B. Output response formulas

The method allows to compute the lossless line output step response comply the losses parameter ε. The way of calculating the step output response for such a system is presented in [6]. For the first traveling wave we obtain:

$$v_{out}(d,t) = v_{01}(d,t) + v_{11}(d,t) =$$
$$= \frac{E_0}{\beta+1}\left(A\cdot\left(\frac{t-T}{T}\right)\cdot e^{-\frac{\alpha}{T}(t-T)} - B\cdot\left(1 - e^{-\frac{\alpha}{T}(t-T)}\right)\right)\cdot \mathbf{1}(t-T), \tag{6}$$

for time 0<t<3T

where:

$$A = -\varepsilon\cdot\alpha\cdot e^{-1.5\varepsilon},$$
$$B = \frac{\varepsilon}{2}e^{-0.5\varepsilon}\left(1+e^{-\varepsilon}\right) - \frac{\beta\varepsilon}{\beta+1} - 2e^{-0.5\varepsilon}.$$

The simulations show that the approximation gives good results, especially during the rise time of output signal.

The ramp output response:

$$v_{out}^r(t) = \frac{E_0}{(\beta+1)T_r}\cdot$$
$$\cdot\left[k\left(1 - e^{-\alpha\frac{t-T}{T}}\right) - \frac{t-T}{T}\left(B + \frac{A}{\alpha}e^{-\alpha\frac{t-T}{T}}\right)\right], \tag{7a}$$

for $t - T \le T_r$

$$v_{out}^r(t) = \frac{E_0}{(\beta+1)T_r}\cdot\left[\begin{matrix}\left(\left(\dfrac{t-T-T_r}{T\alpha}\right)A \atop +k\right)e^{-\alpha\left(\frac{t-T_r-T}{T}\right)} \\ -\left(\dfrac{t-T}{T\alpha}A + k\right)e^{-\alpha\frac{t-T}{T}} - \dfrac{T_r}{T}B\end{matrix}\right], \tag{7b}$$

for $t - T \ge T_r$

where $k = \left(\dfrac{A}{\alpha^2} + \dfrac{B}{\alpha}\right)$.

C. Threshold crossing time calculation

The computation of threshold crossing time is a challenge many authors try to deal with e.g. [6]. For the ramp response (7) we can calculate the threshold crossing time, during the ramp times (t<Tr) solving (8) with respect to t:

$$\rho = \frac{1}{T_r(\beta+1)}\Big[K_1T_2\tilde{t}e^{-\alpha\tilde{t}} + C_2T_r\tilde{t} + -e^{-\alpha\tilde{t}}\left(K_2T_2 - K_1T_re^{\alpha T_r}\right) - 0.5C_2T_r^2 - B_2T_r\Big] \tag{8}$$

where ρ is the threshold value (0÷1).
Equation (11) can be reduced to the form

$$a_{r2} - K_r z = z e^z, \tag{9}$$

where a_{r2} and K_r are constants containing combination of RLC and R_w, C_0, ρ and T_r parameters, and z is the time function.
To solve the equation (9) and obtain the threshold crossing time we approximate Lambert function, occurring in solution of (9) and calculate the threshold time iteratively. For the ramp response the threshold crossing time is given by:

$$\tau_{\rho r}^{(n)} = \left(-\frac{1}{\alpha}z^{(n)} - T_4\right)\cdot T \tag{10}$$

where the T_4 is a constant dependent on interconnect parameters and the z is given by an iterative procedure:
$z^{(n+1)} = W\left(a_{r2} - K_r z^{(n)}\right)$, and W(x) is Lambert W function.

III. The sensitivity calculations

The interconnect response both for step and ramp response can be highly sensitive to small changes in geometrical parameter values Fig.1. Due to generation by the presented approach the closed form formulas for output response (7) and (8), we could calculate the sensitivity of the output response *v* to parameter *λ* with the formula:

$$S_v^\lambda = \frac{\lambda}{v}\frac{\partial v}{\partial \lambda} \tag{11}$$

The sensitivity of the formula (11) is compared with the sensitivity of the SPICE simulations for the RLC parameter analysis. The sensitivity in SPICE program we compute using the output signal with perturbed parameter to which the sensitivity is calculated. We use the following formula to calculate the sensitivity of the ramp response in SPICE:

$$S_{vSPICE}^\lambda = \frac{v(t,\lambda) - v(t,\lambda\cdot 1.01)}{v(t,\lambda)}100 \tag{1}$$

The sensitivity for the step response obtained from (11) compared with (12) gives very good results for RLC parameters.

Sensitivity of the threshold crossing time to geometrical parameters will become determined similarly as sensitivity of the step and ramp responses.

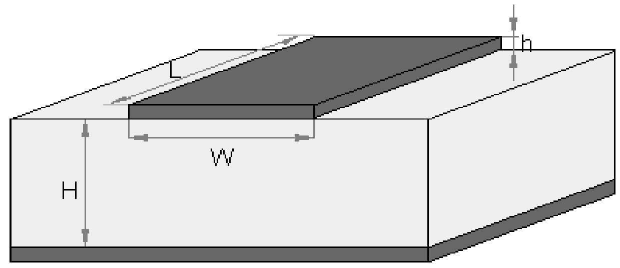

Fig.2. Microstrip geometrical model.

Formulas describing the sensitivity of the threshold crossing time to the width of microstrip W as well as for distance from ground plane H, take the form respectively:

$$S_{t_{\rho r}}^{W}=\frac{dt_{\rho r}}{dW}\frac{W}{t_{\rho r}}=\left(\frac{dt_{\rho r}}{dR}\frac{\partial R}{\partial W}+\frac{dt_{\rho r}}{dL}\frac{\partial L}{\partial W}+\frac{dt_{\rho r}}{dC}\frac{\partial C}{\partial W}\right)\frac{W}{t_{\rho r}} \quad (13)$$

$$S_{t_{\rho r}}^{H}=\frac{dt_{\rho r}}{dH}\frac{H}{t_{\rho r}}=\left(\frac{dt_{\rho r}}{dR}\frac{\partial R}{\partial H}+\frac{dt_{\rho r}}{dL}\frac{\partial L}{\partial H}+\frac{dt_{\rho r}}{dC}\frac{\partial C}{\partial H}\right)\frac{H}{t_{\rho r}} \quad (14)$$

Where derivatives of the threshold crossing time with respect to interconnect parameters are given by:

$$\frac{dt_{\rho r}}{dR}=\frac{1}{Z_0}\frac{\partial t_{\rho r}}{\partial \varepsilon},$$

$$\frac{dt_{\rho r}}{dC}=\frac{\partial t_{\rho r}}{\partial \alpha}\frac{\partial \alpha}{\partial C}+\frac{\partial t_{\rho r}}{\partial \beta}\frac{\partial \beta}{\partial C}+\frac{\partial t_{\rho r}}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial C},$$

$$\frac{dt_{\rho r}}{dL}=\frac{\partial t_{\rho r}}{\partial \alpha}\frac{\partial \alpha}{\partial L}+\frac{\partial t_{\rho r}}{\partial \beta}\frac{\partial \beta}{\partial L}+\frac{\partial t_{\rho r}}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial L},$$

and

$$\frac{\partial \alpha}{\partial C}=\frac{1}{C_0},\quad \frac{\partial \beta}{\partial C}=\frac{R_w}{2\sqrt{LC}},\quad \frac{\partial \varepsilon}{\partial C}=\frac{R}{2\sqrt{LC}},$$

$$\frac{\partial \alpha}{\partial L}=0,\quad \frac{\partial \beta}{\partial R}=-\frac{R_w}{2L}\sqrt{\frac{C}{L}},\quad \frac{\partial \varepsilon}{\partial L}=-\frac{R}{2L}\sqrt{\frac{C}{L}}.$$

However derivative of interconnect parameters RLC with respect to geometrical variables are:

$$\frac{\partial R}{\partial W}=-\frac{\rho\cdot d}{W^2 h},\quad \frac{\partial R}{\partial H}=0$$

$$\frac{\partial L}{\partial W}=\left[5.0832\cdot 10^{-9}d\cdot\left(W+0.4h\left(\ln\left(\frac{12.57W}{h}\right)+1\right)\right)^2\cdot\right.$$
$$\left.\cdot\left(\frac{0.4h}{W}+1\right)-32h^2\right]\Bigg/\left[0.4h\left(\left(\ln\left(\frac{12.57W}{h}\right)+1\right)\right)\cdot\right.$$
$$\left.\cdot\left(W+0.4h\left(\ln\left(\frac{12.57W}{h}\right)+1\right)\right)^2++32h^2+W\right],$$

$$\frac{\partial L}{\partial H}=\left[5.0832\cdot 10^{-9}d\cdot\left(\frac{8}{W+\frac{1.25h}{\pi}\left(1+\ln\frac{4\pi W}{h}\right)}+\right.\right.$$
$$\left.\left.-\frac{W+\frac{1.25h}{\pi}\left(1+\ln\frac{4\pi W}{h}\right)}{4H^2}\right)\right]\Bigg/$$
$$\Bigg/\left[\frac{8H}{W+\frac{1.25h}{\pi}\left(1+\ln\frac{4\pi W}{h}\right)}+\frac{W+\frac{1.25h}{\pi}\left(1+\ln\frac{4\pi W}{h}\right)}{4H}\right],$$

$$\frac{\partial C}{\partial W}=\frac{\varepsilon}{H},$$

$$\frac{\partial C}{\partial H}=\varepsilon\left[-\frac{W-h/2}{H^2}-\frac{2\pi\left(\frac{2}{h}\sqrt{\frac{2H}{h}\left(\frac{2H}{h}+2\right)}+\frac{4H}{h^2}+\frac{2}{h}\right)}{M}\right],$$

for $W\geq h/2$ and

$$\frac{\partial C}{\partial H}=-\frac{W}{H^2}-\left[0.0543\frac{h}{2H^2}\ln\left(1+\frac{2H}{h}+\sqrt{\frac{2H}{h}\left(\frac{2H}{h}+2\right)}\right)+\right.$$
$$\left.-\left(\frac{2}{h}\sqrt{\frac{2H}{h}\left(\frac{2H}{h}+2\right)}+\frac{4H}{h^2}+\frac{2}{h}\right)\cdot\pi\cdot\left(1-0.0543\frac{h}{2H}\right)\right]\frac{1}{M},$$

for $W<h/2$,

where:

$$M=\left(\sqrt{\frac{2H}{h}\left(\frac{2H}{h}+2\right)}\left(1+\frac{2H}{h}\right)+\left(\frac{4H^2}{h^2}+\frac{4H}{h}\right)\right)\cdot$$
$$\cdot\ln\left(1+\frac{2H}{h}+\sqrt{\frac{2H}{h}\left(\frac{2H}{h}+2\right)}\right)^2.$$

The formulas for the derivatives $\partial t_{\rho r}/\partial\alpha$, $\partial t_{\rho r}/\partial\beta$, $\partial t_{\rho r}/\partial\varepsilon$ are rather complicated and will be not quoted here, however can be found by differentiation of e.g. (8).

IV. The simulation results

Sensitivity calculations, by means of formula quoted in the previous section, are compared to the sensitivities calculated using formula (12) and simulations performed in the PSPICE program. The results are presented in the Figs 3-6.

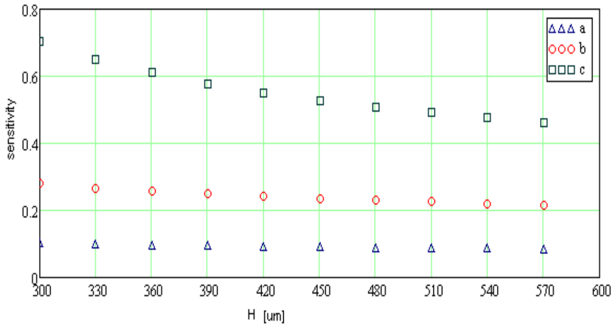

Fig. 3. Comparison of the sensitivity of crossing threshold time to the microstrip parameter H for various microstrip lengths and distances H for exemplary interconnect parameters. (ρ=0.5, number of iterative N=10) a - C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=2mm, b- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=3mm, c- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=3mm, d- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, H=300µm, d=4mm.

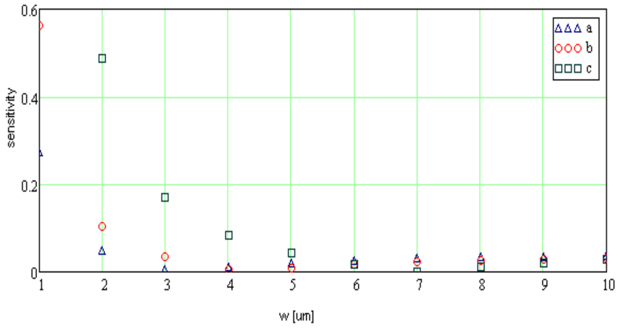

Fig. 4. Comparison of the sensitivity of crossing threshold time to the microstrip width W for various load capacitances and the microstrip widths W for exemplary interconnect parameters. (ρ=0.5 number of iterative N=10) a - C_0=0.5pF and R_w=12.5Ω, H=300µm, W=2µm, h=1µm, d=2mm, b- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=2mm, c- C_0=0.5pF and R_w=50Ω, W=2µm, h=1µm, d=3mm, d- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, H=300µm, d=2mm.

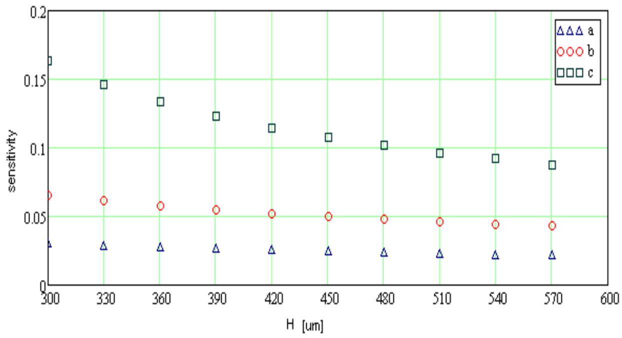

Fig. 5. Comparison of the sensitivity of crossing threshold time to the distance H for various microstrip lengths and distances H between microstrip and the ground for exemplary interconnect parameters. (ρ=0.5 number of iterative N=10) a - C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=2mm, b- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=2mm, c- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=3mm, d- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, H=300µm, d=4mm.

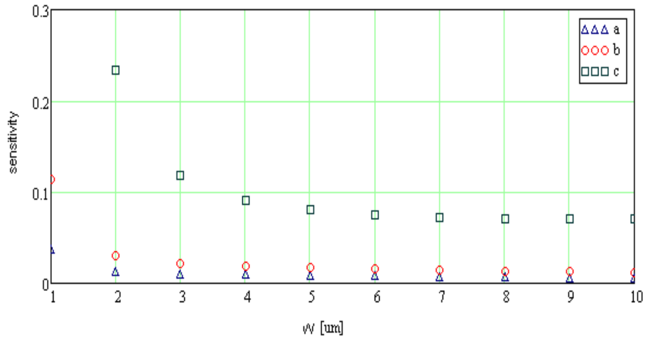

Fig. 6. Comparison of the sensitivity of crossing threshold time to the distance H for various load capacitance and microstrip widths W, for exemplary interconnect parameters. (ρ=0.5 number of iterative N=10) a - C_0=0.5pF and R_w=12.5Ω, H=300µm, W=2µm, h=1µm, d=2mm, b- C_0=0.5pF and R_w=25Ω, W=2µm, h=1µm, d=2mm, c- C_0=0.5pF and R_w=50Ω, W=2µm, h=1µm, d=3mm

V. CONCLUSIONS

Presented above relationships permits for analytical calculation of sensitivity in low loss interconnects to the geometrical such as microstrip width and thickness of the substrate. The presented method of calculation of the sensitivity could be easily extended to sensitivity calculation of the threshold crossing time to the remaining geometrical material parameters. The exactness of the method depends if the interconnect can be considered as low loss ($R<1/2Z_0$) and on exactness of the formula for RLC parameters as a function of the geometrical and material data.

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