Step response sensitivity of VLSI Interconnects

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***Abstract*— In the paper sensitivity of the voltage step response of the system inverter-interconnect-inverter with respect to selected parameters is considered. The sensitivity coefficients for normalized parameters are given. These formulas correspond with sensitivity coefficients to interconnect RLC parameters and input and output parameters. The sensitivity is calculated directly from step response formula. The closed form formula for the output response allows to present closed form formulas the for sensitivity.**

Keywords—VLSI interconnect, RLC transmission line, sensitivity, step response

I. INTRODUCTION

The VLSI analysis and modeling need to take into account the VLSI interconnects. Faster circuits, make the interconnect the critical point in circuit analysis. For higher level interconnects the RLC transmission line model must be used to model the interconnects.

Sensitivity characterizes the influence of changing parameter value on system functionality. Sensitivity analysis permits to estimate, which of system parameters affect more and which one of them less on it. It is also possible to identify the number of variables, which have critical influence on operation of the circuit. There is a problem to calculate the sensitivity of the threshold crossing time when the exact formula is not derived. The problem of sensitivity calculation is considered e.g. in [1,2].

The step response calculation for higher level wires assuming RLC transmission model and the global resistance less than lossless interconnect impedance is presented in [3]. In this point we will give only an outline of this method. First for the low resistance interconnect we can write:

$$\frac{R}{2Z_c} \leq 1$$

That assumption allows treat the line as low-loss line with high inductance influence. The considered system is shown in Fig.1. and the transmission line is electrically long.

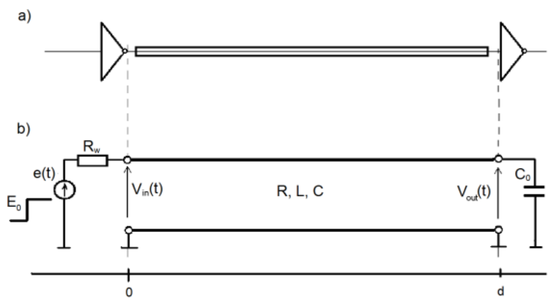

Fig. 1. a) The considered system inverter-interconnect-inverter, b) the model of the considered system

The work is organized as follows. In next section there are presented the foundations of step response calculation. In third section the sensitivity calculations are presented. We conclude in the next section

II. SCATTERING PARAMETERS CALCULATION

General information

Instead of voltage–current, we can use the forward and backward current waves as the dependent variables to describe the transmission line [1]. Then the current waves are defined as follows:

$$\begin{aligned} i_-(y,\tau) &= \frac{1}{2}\left(Y_c(y)u(y,\tau) - i(y,\tau)\right) \\ i_+(y,\tau) &= \frac{1}{2}\left(Y_c(y)u(y,\tau) + i(y,\tau)\right) \end{aligned} \qquad (1)$$

Where

i *and* u - current, voltage along the line

y - space coordinate normalized in relation to the line length *l*

τ - temporal coordinate normalized in relation to the line delay

$Y_c(y)$ - characteristic admittance of the line

Calculation of scattering parameters requires introducing normalized current waves in the following form:

$$
\begin{aligned}
i_-^n(y,\tau) &= i_-(y,\tau)\sqrt{Z_c(y)} \\
i_+^n(y,\tau) &= i_+(y,\tau)\sqrt{Z_c(y)}
\end{aligned} \qquad (2)
$$
,

$Z_c(y)$ - characteristic impedance of the line

Which are related to incident and reflected power waves a1,a2, b1, b2.

$$
\begin{aligned}
i_-^n(0,\tau) &= b_1(\tau) \\
i_-^n(1,\tau) &= a_2(\tau) \\
i_+^n(0,\tau) &= a_1(\tau) \\
i_+^n(1,\tau) &= b_2(\tau)
\end{aligned} \qquad (3)
$$
,

In scattering parameters calculation we take into account the load and input as characteristic impedances, as presented on the picture (Fig. 2.):

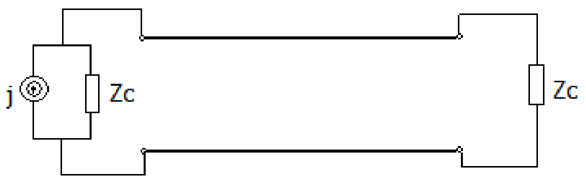

Fig.2. The model of interconnect.

The exact form for the scattering parameters in Laplace domain can be expressed as follows:

$$
\begin{aligned}
S_1(p) &= \frac{F_2(p)}{1+F_1(p)} \frac{1-e^{-2\Gamma(p)}}{1+\rho(p)e^{-2\Gamma(p)}} \\
S_2(p) &= \frac{1}{1+F_1(p)} \frac{2e^{-2\Gamma(p)}}{1+\rho(p)e^{-2\Gamma(p)}}
\end{aligned} \qquad (4)
$$
,

where:

$$
F_1(p) = \frac{p+0.5\varepsilon}{\Gamma(p)}, \quad F_2(p) = \frac{-0.5\varepsilon}{\Gamma(p)},
$$

$$
\Gamma(p) = \sqrt{(p+\varepsilon)p},
$$

$$
\rho(p) = \frac{\Gamma(p)-p-\varepsilon}{\Gamma(p)-0.5\varepsilon}, \quad \varepsilon = \frac{R_t}{Z_c},
$$

and R_t is total resistance of the line

The proposed solution allows to calculate the time domain scattering parameters. The scattering parameters in the Laplace domain are approximated by the method of successive approximations. Then we obtain the first approximation of the form:

$$
\begin{aligned}
S_{1\_1}(p) &= \frac{\beta}{2} \frac{1}{p+\alpha}\left(e^{-2(p+\alpha)}-1\right), \\
S_{2\_1}(p) &= e^{-(p+\alpha)},
\end{aligned} \qquad (5)
$$

where $\beta = \frac{R_w}{Z_c}$, $\alpha = \frac{C_t}{C_0}$,

R_w is output resistance of input gate,

C_0 is input capacitance of output gate,

C_t is total capacitance of the line.

Although the first approximation gives good results for both module and phase of S_1 and phase of S_2, the module of S_2 is not approximated properly. Then we calculate the second approximation of S_2 of the form:

$$
\begin{aligned}
S_{2\_2}(p) = e^{-(p+\alpha)} \cdot \left(1 + \left(\frac{\alpha}{2(p+\alpha)}\right)^2 (2(p+\alpha)-1)\right) + \\
+ \left(\frac{\alpha}{2(p+\alpha)}\right)^2 (2(p+\alpha)-1) e^{-3(p+\alpha)}.
\end{aligned} \qquad (6)
$$

After second approximation is added the results are much better, but the form of the parameter is more complicated.

Step response of loaded interconnect

The single interconnect driven by a gate and loaded by a gate can be modeled as transmission line with input resistance and output capacitance. The system of equation for RLC transmission line take the form:

$$
\begin{cases}
-\dfrac{\partial v}{\partial x} = Ri + L\dfrac{\partial i}{\partial t}, \\
-\dfrac{\partial i}{\partial x} = C\dfrac{\partial v}{\partial t},
\end{cases}
$$

$$
i(x,0) = 0,
$$

$$
v(x,0) = 0
$$

$$
e(t) - R_w i(0,t) = v(0,t),
$$

$$
i(d,t) = C_0 \frac{\partial v(d,t)}{\partial t}.
$$

The step response of such model can be written as:

$$
u(t) = E \cdot \int_0^t h(\tau)d\tau \qquad (7)
$$

and the h(t) we obtain from inverse Laplace transformation of H(p) written as:

$$
H(p) = \frac{Z_c}{R_w+Z_c} \frac{(1+\rho_0)\cdot S_2}{S_2 \cdot \left(\frac{(\rho_w+\rho_0)}{(\rho_w \cdot \rho_0)}\right) - S_1^2 + S_2^2} \qquad (8)
$$

Where ρ_w and ρ_0 are input and output reflection coefficients respectively.

We can rewrite the formula to the form:

$$
H(p) = \frac{2\alpha e^{-\frac{\varepsilon}{2}-p}}{(\beta+1)(\alpha+p)} \sum_k (q)^k \qquad (9)
$$

Where:

$$q=\frac{\varepsilon\left(e^{-\varepsilon-2p}-1\right)\left(p-\beta\alpha\right)}{\left(\varepsilon+2p\right)\left(\beta+1\right)\left(\alpha+p\right)}-\frac{\varepsilon^{2}\left(e^{-\varepsilon-2p}-1\right)^{2}\left(\alpha-p\right)\left(\beta-1\right)}{4\left(\varepsilon+2p\right)^{2}\left(\beta+1\right)\left(\alpha+p\right)}+$$
$$+\frac{e^{-\varepsilon-2p}\left(\alpha-p\right)\left(\beta-1\right)}{\left(\beta+1\right)\left(\alpha+p\right)}$$

For the threshold time calculation, and sensitivity of step response in rise time analysis we take into account T<t<3T, which for the high inductive interconnect are typically sufficient:

$$v_s\left(t\right)=Ee^{-\frac{\varepsilon}{2}}\cdot\left[I_a\left(t-1\right)e^{-\frac{\varepsilon(t-1)}{2}}+\right.$$
$$\left.+I_b\left(t-1\right)e^{-\alpha(t-1)}+I_c\left(t-1\right)\right]\mathbf{1}\left(t-1\right) \tag{10}$$

$$I_a\left(t\right)=\frac{-\alpha}{\left(\varepsilon-2\alpha\right)^2\left(\beta+1\right)^2}\left[8\beta\alpha+4\varepsilon+\right.$$
$$\left.+\frac{\left(\beta-1\right)}{2\left(\varepsilon-2\alpha\right)}\cdot\left(\begin{array}{l}\left(12\alpha\varepsilon-8\alpha^2+4\varepsilon^2\right)\\+t\cdot\varepsilon\left(\varepsilon^2-4\alpha^2\right)\end{array}\right)\right]\mathbf{1}\left(t\right) \tag{11}$$

$$I_b\left(t\right)=\frac{2}{\left(\beta+1\right)}+\frac{1}{\left(\varepsilon-2\alpha\right)^2\left(\beta+1\right)^2}$$
$$\left[\left(4\varepsilon\alpha-2\beta\varepsilon^2+8\beta\alpha\varepsilon\right)+\right.$$
$$\left.-\frac{\left(\beta-1\right)\varepsilon^2\left(\varepsilon-10\alpha\right)}{2\left(\varepsilon-2\alpha\right)}+t\alpha\right]\mathbf{1}\left(t\right) \tag{12}$$

$$I_c\left(t\right)=\frac{2}{\left(\beta+1\right)}+\frac{1}{\left(\varepsilon-2\alpha\right)^2\left(\beta+1\right)^2}\cdot$$
$$\left[-t\left(2\alpha\varepsilon^2-4\varepsilon\alpha^2+2\beta\alpha\varepsilon^2-4\beta\alpha^2\varepsilon\right)\right.$$
$$+\frac{\left(\beta-1\right)\left(12\alpha^2\varepsilon-8\alpha^3-6\alpha\varepsilon^2+\varepsilon^2\right)}{2\left(\varepsilon-2\alpha\right)}$$
$$\left.-\left(8\beta\alpha^2-8\beta\alpha\varepsilon+2\beta\varepsilon^2\right)\right]\mathbf{1}\left(t\right) \tag{13}$$

The simulation results are presented in chapter fourth.

III. SENSITIVITY CALCULATIONS

The output response can be highly sensitive to small changes in the parameter values. Due to generation, by the presented approach, the closed form formula for output response (10), we can calculate the sensitivity of the output response with the formula:

$$S_v^\lambda=\frac{\lambda}{v}\frac{\partial v}{\partial\lambda} \tag{14}$$

where λ is a parameter with respect to which sensitivity is calculated.

The sensitivity to parameters RLC and output and input parameters C_0 and R_s are simply dependent on sensitivity of normalized parameters ε, β, α. The dependence can be written as:

$$S_{v_s}^R=\frac{1}{Z_0}v'_{s\varepsilon}\frac{R}{v_s}=\frac{\beta}{v_s}v'_{s\varepsilon}=S_{v_s}^\varepsilon \tag{15}$$

$$S_{\mathrm{v}_s}^L(t)=-\frac{1}{2}\left(S_{v_s}^\beta(t)+S_{\mathrm{v}_s}^\varepsilon(t)\right)-\frac{\mathrm{v}'_{s\tilde{t}}}{\mathrm{v}_s}\frac{\mathrm{t}}{2\sqrt{LC}} \tag{16}$$

$$S_{\mathrm{v}_s}^C(t)=S_{v_s}^\alpha(\mathrm{t})+\frac{1}{2}\left(S_{v_s}^\beta(t)+S_{\mathrm{v}_s}^\varepsilon(t)\right)-\frac{\mathrm{v}'_{s\tilde{t}}}{\mathrm{v}_s}\frac{\mathrm{t}}{2\sqrt{LC}} \tag{17}$$

$$S_{v_s}^{R_w}=\frac{1}{Z_0}v'_{s\beta}\frac{R_w}{v_s}=\frac{\beta}{v_s}v'_{s\beta}=S_{v_s}^\beta \tag{18}$$

$$S_{v_s}^{C_0}=-\frac{C}{C_0}\frac{v'_{s\alpha}}{v_s}=-\alpha\frac{v'_{s\alpha}}{v_s}=S_{v_s}^\alpha \tag{19}$$

where v'_{sx} is a derivative of the voltage output response ($u_s(t)$) to parameter x.

The sensitivity to parameter α can be easily calculate and is given by:

$$S_{v_s}^\alpha\left(\tau\right)=\alpha\frac{\left(I_{a\alpha}\left(\tau\right)e^{-\frac{\varepsilon\tau}{2}}+I_{b\alpha}\left(\tau\right)e^{-\alpha\tau}-\alpha I_b\left(\tau\right)e^{-\alpha\tau}+I_{c\alpha}\left(\tau\right)\right)}{\left(I_a\left(\tau\right)e^{-\frac{\varepsilon\tau}{2}}+I_b\left(\tau\right)e^{-\alpha\tau}+I_c\left(\tau\right)\right)} \tag{20}$$

where I_{ax} is a derivative of I_a with respect to parameter x and τ = (t -T)/T.

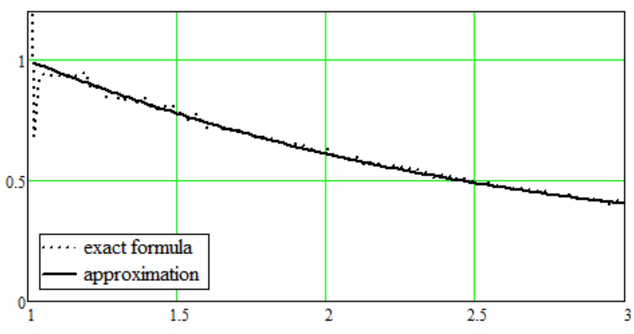

Fig. 3. Sensitivity of step response to α for an interconnect for exemplary parameters: R_t=25Ω, L_t=10nH, C_t=1pF, R_s=25Ω, C_0=0.1pF,

The sensitivity to parameter β can be easily calculate and is given by:

$$S_{v_s}^{\beta}(\tau)=\beta\frac{\left(I_{a\beta}(\tau)e^{-\frac{\varepsilon\tau}{2}}+I_{b\beta}(\tau)e^{-\alpha\tau}+I_{c\beta}(\tau)\right)}{\left(I_a(\tau)e^{-\frac{\varepsilon\tau}{2}}+I_b(\tau)e^{-\alpha\tau}+I_c(\tau)\right)} \tag{21}$$

The sensitivity to parameter ε is given by:

$$S_{v_s}^{\varepsilon}(\tau)=\varepsilon\frac{\left(I_{a\varepsilon}(\tau)e^{-\frac{\varepsilon\tau}{2}}-\frac{\tau}{2}I_a(\tau)e^{-\frac{\varepsilon\tau}{2}}+I_{b\varepsilon}(\tau)e^{-\alpha\tau}+I_{c\varepsilon}(\tau)\right)}{\left(I_a(\tau)e^{-\frac{\varepsilon\tau}{2}}+I_b(\tau)e^{-\alpha\tau}+I_c(\tau)\right)} \tag{22}$$

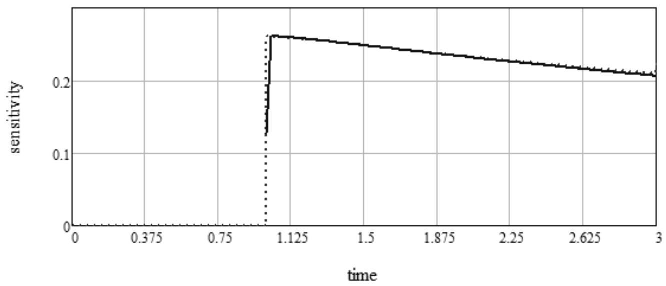

Fig. 4. Sensitivity of step response to β for an interconnect for exemplary parameters: R_t=25Ω, L_t=5nH, C_t=1pF, R_s=25Ω, C_0=0.1pF,

The comparison of the sensitivity from the formulas 20-22 with the sensitivity calculated in SPICE are presented on figures 3-4. The result of analytical calculations gives quite good result for the low loss interconnects and is fast and efficient. The analytical form of the result allows to use it in other calculation without doing simulations for each set of parameters.

The sensitivity for some parameter changes are given on Figures 5-7.

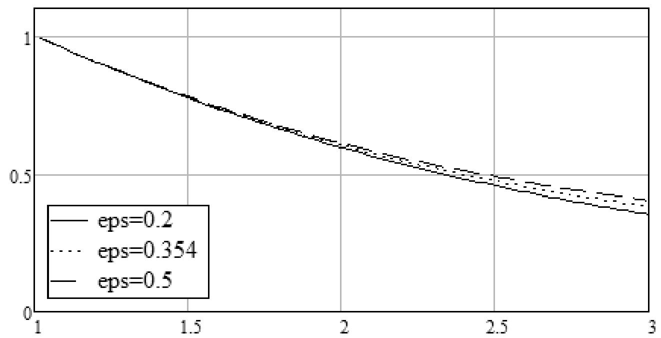

Fig. 5. Sensitivity of step response to α for an interconnect for different losses value (α=1, β=1)

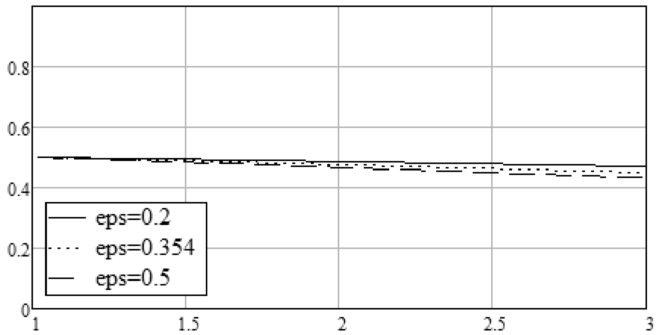

Fig. 6. Sensitivity of step response to β for an interconnect for different losses value (α=1, β=1)

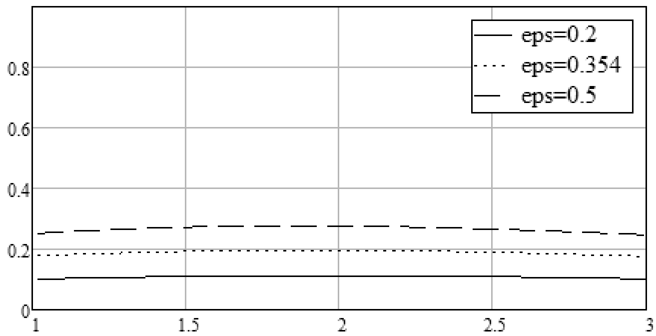

Fig. 7. Sensitivity of step response to ε for an interconnect for different losses value (α=1, β=1)

IV. Conclusions

In the paper we present the analytical method of sensitivity calculation. First we calculate the step response for VLSI interconnect. The higher level interconnects characterize the high inductance compared to resistance, than we can use several approximation to obtain the analytical formula. As we present the results are good enough to $t_{50\%}$ calculation or signal shape observation.. Although the SPICE calculation are always possible, the analytical form of the sensitivity allows to obtain very fast and efficient calculations.

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