Wideband Spectrum Sensing Utilizing Cumulative Distribution Function and Machine Learning

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***Abstract*—Blind spectrum sensing (BSS) is a valuable technique for identifying unknown signals in scenarios where prior knowledge is limited. However, traditional methods encounter difficulties when dealing with unknown and time-varying signals in the presence of noise. This paper addresses these challenges by enhancing machine learning (ML) features through a novel statistical signal processing approach. The proposed BSS approach integrates cumulative distribution functions (CDFs) into an unsupervised ML process, allowing for the effective clustering of distinct transmission states without making assumptions about specific noise distributions. Furthermore, the paper introduces a temporal decomposition technique that utilizes shorter Fast Fourier Transforms (FFTs) to enhance learning, reduce system inertia, and minimize the amount of data required for retraining in changing conditions. Simulation results presented in this paper demonstrate a good detection rate in a generic transmission scenario (i.e., receiving a Gaussian pulse disturbed by additive white Gaussian noise) while maintaining a constant false alarm rate. These findings indicate the efficacy of the proposed BSS approach in handling unknown signals and its potential for practical implementation.**

***Index Terms*—Blind detection, cumulative distribution function, machine learning, spectrum sensing, unknown signals.**

I. Introduction

Blind spectrum sensing (BSS) is a powerful technique that enables the detection of unknown signals in scenarios where little or no prior knowledge about the signal is available. Traditional detection methods may prove ineffective in scenarios where the signals are unknown, vary over time, or are surrounded by significant levels of noise and interference. One of the significant challenges of blind detection is the need for large amounts of data, computational expense, and less accurate results compared to methods with prior knowledge of the signal. However, the ability to detect unknown signals can be highly valuable in a wide range of applications.

Wireless communications and cognitive radio networks require the detection and tracking of signals from multiple sources to avoid harmful interference [1]. In radio astronomy, it is often necessary to detect unknown signals from distant sources [2]. Passive radar systems rely on naturally occurring signals, such as TV or FM signals, as the source of illumination rather than an active transmitter [3]. In medical imaging, it is necessary to detect and locate small or faint signals within a large amount of data. The above-listed detection constraints can also be encountered in different situations.

A diverse range of advanced and well-established techniques have been successfully applied across this interdisciplinary domain. Among these, subspace methods are widely used for blind spectrum sensing. Principal component analysis (PCA) and independent component analysis (ICA) are two popular subspace techniques that aim to identify a low-dimensional subspace that captures the most essential features of the signal [4]. Although subspace techniques are efficient in separating mixed or correlated signals and are relatively easy to implement, they require large amounts of data and can be sensitive to noise and outliers. Cyclostationary feature detection is another technique that can be used for blind detection. This method relies on the cyclostationarity property of signals, which refers to the statistical dependence between the signal and its time-shifted versions. Cyclostationary feature detection is known for its robustness to noise and interference, but it applies only to a limited range of signal types, such as narrowband signals [5]. Sparse representations, a recent technique for BSS, including compressive sensing, aim to represent the signal as a sparse linear combination of basis functions. Sparse representations can effectively extract meaningful information from signals, even in the presence of noise and interference. However, they require a significant amount of data and can be affected by the selection of basis functions used [6].

In recent years, the field of BSS has seen a surge of interest in developing novel algorithms and techniques to improve the performance and robustness of signal detection. Researchers have focused on several areas of investigation, such as the development of new subspace-based techniques and sparse representation methods. Notably, Machine Learning (ML) approaches have garnered considerable attention as well [7]–[11]. These methods employ ML algorithms that can adapt to the signal and noise characteristics. However, two significant challenges remain open in ML-based BSS research:

1) Development of versatile ML features that can handle non-stationary and non-linear signals independently of their characteristics is essential. Such features should enable the detection of unknown signals without prior knowledge of their frequency, modulation, or structure and adapt to signals with time-varying characteristics.
2) Requirement of large amounts of data for achieving high

accuracy using machine learning-based methods can be a prohibitive factor, especially in scenarios with limited data or computationally intensive detection tasks.

Overcoming these challenges is critical for the advancement of BSS utilizing ML techniques.

The aim of this paper is to address the challenges identified in ML-based BSS. To this end, we extend the available range of ML features with a novel statistical signal-processing method. By integrating cumulative distribution functions (CDFs) into the ML process and utilizing unsupervised ML, the proposed model effectively discriminates between statistically distinct states without assuming a specific noise distribution. The temporal decomposition technique enhances learning by employing multiple shorter FFTs within a single time frame, reducing system inertia and minimizing data requirements for model retraining under changing propagation conditions.

The remainder of the paper is structured as follows: Section II introduces the ML-based Blind Spectrum Sensing (BSS) system, providing an overview of the commonly utilized measurements and features. In Section III, the system model and the statistical foundation for CDF-based detection are introduced. Section IV introduces the CDF-based approach to ML-based detection, presenting simulation results concerning CDF measurements in BSS and discussing the accomplished enhancements through the utilization of CDF-based techniques. Finally, Section VI presents the conclusion, summarizing the obtained findings and outlining potential avenues for future research.

II. Machine learning in BSS

Machine learning has emerged as a powerful technique for signal detection in recent years [7]–[11]. These methods aim to extract meaningful information from signals, even in noisy environments, without prior knowledge of the signal's characteristics, by learning the underlying patterns in the data. Various features can be extracted from the received signal to classify the presence of an unknown signal in the noise. The most common features used in ML for spectrum sensing include power spectral density, autocorrelation, and cyclostationary features such as cyclic correlation and cyclic spectral density. Other popular features are based on signal statistics, such as the mean, variance, skewness, and kurtosis of the received signal. Additionally, machine learning algorithms may use time-domain features such as energy, entropy, and correlation coefficient between different subcarriers. The choice of features depends on the specific application and the characteristics of the signal being sensed. In some cases, a combination of multiple features is used to improve the algorithm's accuracy and handle the non-stationary and non-linear nature of the signals [8]. A good overview is given by domain-specific surveys like [7], [8].

ML algorithms enhance the decision-making capabilities of detectors by learning from past experiences and through efficient information inference. In [9], unsupervised and supervised learning methods are combined to perform spectrum sensing under multiple transmit powers. The approach first discovers the transmission patterns and statistics through a learning phase based on a modified K-means algorithm and then distinguishes energy feature vectors using support vector machines (SVM). In [11], a random forest spectrum sensing algorithm is introduced for signal recognition in low signal-to-noise ratio (SNR) environments. The proposed method extracts the characteristic parameters with the largest extraction energy and uses the mean and variance of the signal cycle spectrum to classify and detect the deterministic signals. Supervised and semi-supervised learning algorithms were also developed in [10], where the eigenvalues of the received signal covariance matrix are utilized as features. Given the received signal energy and likelihood ratio test statistic with different SNRs, a binary classification-based artificial neural network (ANN) was adopted in [12]. In [13], both the received signal power and cyclic prefix-induced correlation are used as features. A wideband detection scheme is proposed in [14] that improves the detection performance by exploiting the regression and compressive sampling techniques.

To further enhance ML-based detection, three approaches are considered:

1) In [15], an ML-based cooperative sensing algorithm is introduced. The classifier is initially trained on a set of samples containing energy test statistics along with their corresponding decisions about the presence or absence of the signal, and then it is used to predict the decision against the new samples with new energy test statistics. In [16], an SVM-based model for cooperative sensing is proposed that utilizes the signals grouping methodology to decrease the cooperation overhead and enhance the spectrum sensing performance.
2) A hybrid model combines different modeling approaches, such as physical models, data-driven models, or rule-based models, to capture the strengths of each and produce more accurate and reliable results. This can lead to a better understanding of the system and improved decision-making. One example of a hybrid solution is the energy/entropy-based model developed in [17]. Energy detection plays a crucial role in physical-driven detection, supported by entropy as a data-driven approach, both feed as feature vectors to the classifier.
3) Important aspect of hybrid modeling is the decision function. Paper [18] highlights the importance of incorporating decision-making into the modeling process. This involves identifying the optimal solutions based on the model outputs and taking into account various factors, such as cost, feasibility, and environmental impact. In [19], several ML techniques, including SVM, random forest, decision tree, K-NN, logistic regression, and NBC, are trained, validated, and tested.

Up to this point, it should be noted that although integrating ML approaches into hybrid solutions utilizing traditional spectrum sensing features, such as energy/covariance analysis, can yield substantial improvements in spectrum sensing performance, such methodology can come at the cost of increased processing time and implementation complexity. Developing accurate and reliable spectrum sensing models typically involves handling large amounts of data generated from spectrum measurement campaigns. As a result, designers of such models must carefully consider the trade-off between model accuracy and processing efficiency. While the primary focus of this paper is to optimize data utilization, it is important to acknowledge that dimensionality reduction (DR) techniques can also yield substantial benefits by reducing data complexity without compromising accuracy. For a more comprehensive understanding, please refer to [20].

III. System model

Incorporating domain-specific knowledge into detection models is a challenging task, particularly when dealing with complex spectral signals affected by various factors such as fluctuating propagation conditions or frequency-dependent propagation and attenuation. The presence of signal source non-uniformity further complicates the detection process. To address these challenges, statistical feature analysis has emerged as a promising research avenue. This approach avoids relying on predetermined signal parameterization and instead identifies external signals as outliers or rare events within a presumed homogeneous noise environment. The effectiveness of this methodology has been demonstrated in previous studies [21]. Moreover, [22] demonstrates the integration of Goodness-Of-Fit-based spectrum sensing into a conventional wideband spectrum sensing scheme, showcasing an accurate technique with a short sensing time. Their study offers empirical evidence that supports the validity of the statistical approach for wideband signals.

Additive white Gaussian noise (AWGN) is a widely employed channel distortion in radio environments due to several reasons. Firstly, radio channels in practical scenarios are susceptible to multiple noise sources, including thermal, atmospheric, and man-made distortions. The central limit theorem postulates that the sum of numerous independent random variables approximates a Gaussian distribution. In a radio environment, multiple noise sources can be assumed to be independent, random variables that contribute to the overall noise on a channel. Consequently, AWGN is an appropriate choice for modeling the overall noise on a radio channel. Secondly, AWGN is a well-defined statistical model that closely approximates several noise sources' characteristics [23]. Additionally, the Gaussian distribution has some convenient mathematical properties, such as being closed under convolution, which makes it easy to analyze the effects of noise on a signal. Lastly, Gaussian noise represents a worst-case scenario for many radio systems. If a communication system can effectively function in the presence of AWGN, it is likely to perform well in the presence of other types of noise sources. Thus, investigating the effects of AWGN facilitates the design of communication systems that exhibit robustness toward a diverse range of noise sources [24], [25].

In the considered system model, we introduce the following variables: L, the number of sampling points; s_n, the signal under investigation; x_n, the deterministic signal to be detected; z_n, complex Gaussian noise with normal distributed iid samples of zero means and variance σ^2. Additionally, we define H_0 and H_1 as the hypotheses corresponding to "no signal transmitted" and "signal transmitted", respectively. The signal mixed with Gaussian noise is expressed as

$$y_n = \begin{cases} z_n, & \text{if } H_0 \\ x_n + z_n, & \text{if } H_1 \end{cases}. \quad (1)$$

To determine the single-sided spectrum of $s(n)$, we first need to apply a Fourier transform to the received signal

$$S(k) = \sum_{n=0}^{L-1} s(n) e^{-j2\pi\frac{k}{L}}, k = 0, 1, 2 ... \frac{L}{2} - 1, \quad (2)$$

next, we can use the fact that $s(n)$ is a mixture of $x(n)$ and $z(n)$, and the properties of the Fourier transform to write

$$S(k) = X(k) + Z(k), \quad (3)$$

where $X(k)$ and $Z(k)$ are the Fourier transforms of $x(n)$ and $z(n)$, respectively. Thus, the kth bin in the power spectrum can be expressed as

$$P(k) = \frac{1}{L} \left((X_R(k) + Z_R(k))^2 + (X_I(k) + Z_I(k))^2 \right). \quad (4)$$

The notation $X_R(k)$ and $X_I(k)$ denotes the real and imaginary components of the signal, respectively. Analogously, the notation $Z_R(k)$ and $Z_I(k)$ represent the real and imaginary parts of the noise, respectively. The probability distribution of spectrum bins in the states H_0 and H_1 can be determined by analyzing the statistical properties of the received signal $s(n)$ under each hypothesis.

Following the considerations/justification derived in [22], consider the Fourier coefficients distribution of AWGN. As a weighted sum of Gaussian random variables, the Fourier coefficient $Z(k)$ for any given frequency bin k will be Gaussian distributed. Therefore, a complex Gaussian noise process $z(n) \sim \mathcal{N}(0, \sigma^2)$ produces complex Gaussian Fourier coefficients $Z(k)$. As $z(n)$ has zero means, the mean of $Z(k)$ also equals zero. It is also known from probability theory that if $Z(i)$ and $Z(j)$ are two independent random variables with variance σ_i^2 and σ_j^2 respectively, then the random variable $Z(i) + Z(j)$ has variance $\sigma_i^2 + \sigma_j^2$. Hence, the variance of $Z(k)$ can be calculated as [22]

$$\operatorname{var}(Z(k)) = \sum_{n=0}^{L-1} |e^{-j2\pi\frac{kn}{L}}| \sigma^2 = L\sigma^2, \quad (5)$$

thus Fourier coefficients in follow distribution $\sim \mathcal{N}(0, L\sigma^2)$, with L being the length of the DFT.

Consequently, we can also state that the k-th power spectrum coefficient $P(k)$ follows a Chi-squared distribution with two degrees of freedom [22], [26], [27], given by

$$\frac{2|Z(k)|^2}{L\sigma^2} \sim \chi_2^2. \quad (6)$$

as a χ_2^2 distribution is defined as the distribution of the sum of the squares of independent standard normal variables, the factor $2/L\sigma^2$ comes from normalizing the real and the imaginary part of the coefficients $Z(k)$ to $\sim \mathcal{N}(0,1)$ [22].

Under H_1, the received signal is a mixture of the signal $x(n)$ and the noise $z(n)$. Since $x(n)$ and $z(n)$ are independent, their Fourier transforms are also independent. Therefore, under H_1, the power spectrum bin follows a non-central Chi-squared distribution $P(k) \sim \chi_2^2(\lambda_k)$ with two degrees of freedom and non-centrality parameter, given by

$$\lambda_k = \frac{1}{\sigma^2} \left| X(k) + Z(k) \right|^2 . \tag{7}$$

The central chi-squared distribution arises from the sum of squared independent standard normal distributions, while the noncentral chi-squared distribution extends this concept to include normal distributions with any mean and variance. This property enables the incorporation of deterministic narrow or wideband signals in noise, resulting in non-centrality. The presence of non-centrality is a crucial foundation for the detection mechanism, as outlined in the subsequent section.

IV. Proposed solution

By referring to the research findings [22], we can gain insights into the effectiveness of BSS for both narrowband and wideband signals in the presence of AWGN, specifically by analyzing the centrality of the chi-squared distribution. To this end, our proposed methodology utilizes detection anchored in control of the cumulative distribution function, which is introduced as a statistical feature in the machine learning-based approach. Although integrating a new feature into the set of machine learning parameters may not pose a considerable challenge, the issue of handling increasing data quantities remains an open question. Prior considerations have involved the aggregation of data from multiple FFT measurement campaigns in generating empirical distributions of individual spectral peaks. Our solution also addresses this problem by minimizing the amount of data required for effective empirical distribution analysis.

A. Algorithmic outline

The detection is based on a single capture of the signal time frame and three assumptions:

- adjacent noise samples, as iid, can be arbitrarily combined without changing the statistical parameters,
- the temporal form of signals can be subsampled without loss in average power,
- the imperfect matching of the sampling frequency and the frequency of the analyzed signal, causes energy leakage into adjacent FFT bins.

Up to this point, the singly captured sample vector $\mathbf{s}$ of length L is transformed according to the following procedure:

1) we determine the decimation coefficient d,
2) we create a measurement matrix $\mathbf{m}$ with dimensions of d x $\lfloor L/d \rfloor$,
3) the rows of $\mathbf{m}$ are filled with samples of $\mathbf{s}$ in a non-overlapping, interleaved pattern:
$$m(i,j) = S\left((j-1)d + i\right), \tag{8}$$
4) proceed d times the $\lfloor L/d \rfloor$-point FFT, i.e., for each row of the matrix $\mathbf{m}$,
5) for row-oriented FFTs matrix determine the column-oriented empirical CDFs,
6) resolve a center of a polyfigure for each CDF curve,
7) evaluate centroids of the CDFs with respect to the demarcation of the points clustered in the ML procedure.

The approach presented in this discussion offers several significant advantages. Firstly, it integrates a straightforward representation of CDF into the ML process. The previous solution [22] tested the goodness-of-fit with the expected distribution's pattern. However, by introducing ML, we only assume that the two states are statistically distinguishable and rely on the unsupervised clustering performed by ML instead.

Furthermore, temporal decomposition allows for the processing of multiple, substantially shorter FFTs, thus, enabling the acquisition of a sufficient CDF representation for both H_0 and H_1 states from a single measurement capture. This methodology also helps reduce the system's inertia and decreases the amount of data needed to relearn the model when faced with changing propagation conditions that force the algorithm's relearning.

B. Simulation

To assess the performance of the proposed algorithm, we have developed a simulation setup capable of generating random occurrences of a deterministic signal in a complex AWGN environment. The analysis focuses on non-overlapping time frames, each consisting of 4096 samples of complex noise. With a probability of 0.5, a complex Gaussian-modulated sinusoidal Radio Frequency (RF) pulse is added to the frame, as depicted in Figure 1a. The clear signal at the output of the transmitter is denoted as Tx, while the received noisy signal is represented as Rx, indicating the real and imaginary parts as in-phase and quadrature-phase, respectively. The remaining parameters are as follows: Rx bandwidth (BW) of 120 MHz, RF pulse fractional bandwidth $\sim \mathcal{N}(0.5, 0.1)$ and center frequency $\sim \mathcal{N}(0.1, 0.01)$ of BW, and the phase position of the pulse within the frame is controlled by a uniform distribution.

Considering weak signals, i.e., with an SNR below 0 dB, as seen in the full-scale FFT (Fig. 1b), there is no evident indication of signal presence. In the proposed solution, however, we decimate a frame into 32 subframes. As a result of this operation, instead of calculating a single 4096-point FFT, for a row-oriented subframe matrix, we calculate 128 times a row-oriented 32-point FFT. Then, a column-oriented CDF is determined, representing the empirical distributions for individual spectral bins (Fig. 2).

The non-centrality (shift) of the CDFs of the spectral bands that carry the signal is distinctly observable. Subsequently, it is necessary to transform each curve into a more manageable

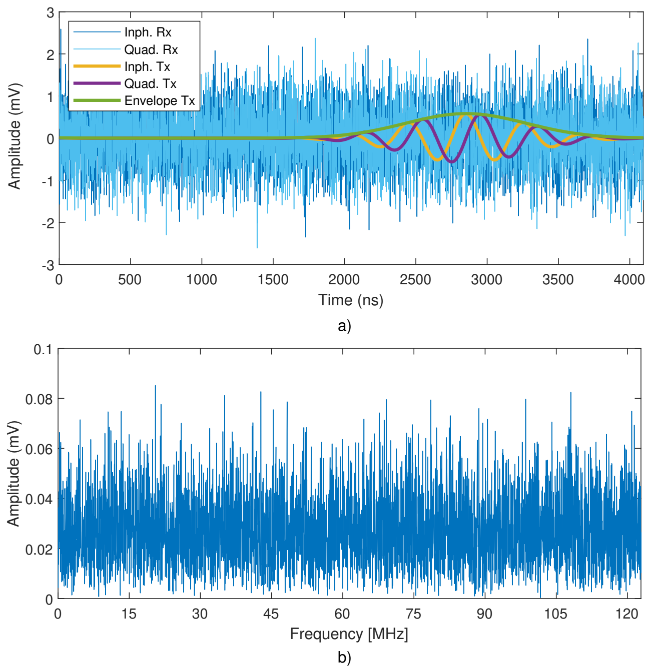

Fig. 1. Single capture of a time frame (a) at the transmitter (Tx) and the receiver (Rx); FFT of the received signal (b).

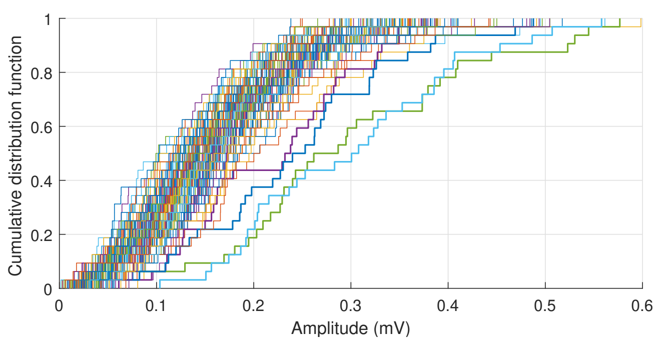

Fig. 2. Column-oriented CDFs of 128 row-oriented 32-point FFTs.

representation. At this stage, the centroid of the polyfigure is determined for each CDF curve (Fig. 3).

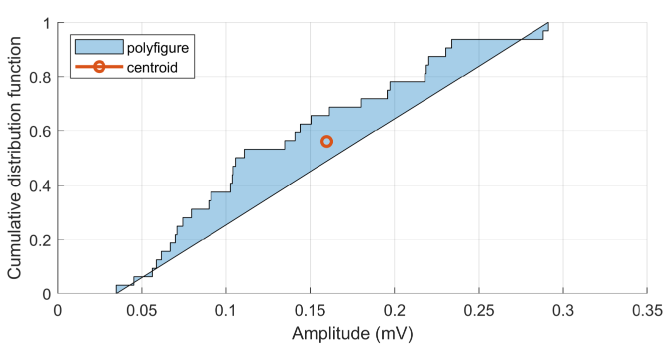

Fig. 3. Polyfigure of a single CDF.

The determined centroids are subjected to unsupervised machine learning for the purpose of separating two groups. At this point, the Cosine Distance is utilized as a similarity measure in the KMeans clustering approach. An exemplary clustering was conducted based on 10 frames generated from a previously described random pattern (Fig. 4). Effective clustering enables the assignment of subsequent points to groups while also allowing for the determination of a demarcation line. In the constant false alarm rate (CFAR) scenario, this line facilitates the adjustment of detector sensitivity.

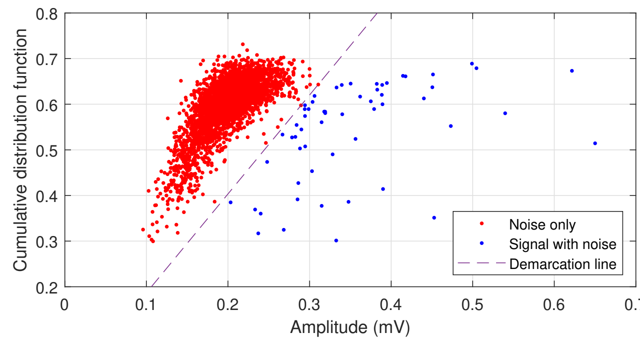

Fig. 4. Cluster assignment.

The performance of the algorithm is measured by the average percentage of correctness in 1000 trials of assigning frames to groups of signal-with-noise or noise-only. Owing to the simulation limitations—i.e., finite number of samples and finite precision of computations—the probability of detection is approximated by the detection rate, while the probability of a false alarm is replaced by the false alarm rate. Figure 5 shows the detection rate in the assumed CFAR scenario, indicating a decrease in the assignment accuracy with a decrease in SNR.

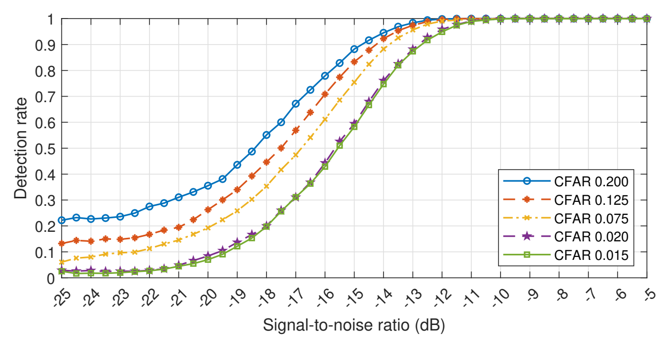

Fig. 5. Detection rate in CFAR scenario.

The obtained results clearly show the usefulness of the proposed approach with regard to even very weak signals, ensuring a 95% detection efficiency down to -12 dB while maintaining a false alarm rate of 0.015.

V. Conclusions

The BSS approach presented in this study offers several significant benefits compared to alternative methods. It incorporates the representation of the CDFs into the ML process and, by leveraging unsupervised ML, effectively distinguishes between two statistically distinct states without assuming a specific noise distribution. The temporal decomposition significantly improves the learning process by utilizing multiple, shorter FFTs from a single time frame capture. Such an approach reduces system inertia and minimizes the data required to retrain the model when encountering changing propagation conditions.

In conclusion, the outcomes obtained through our approach are promising and emphasize the importance of ongoing research and exploration. It is crucial to contextualize our findings within specific scenarios, such as the identification of BPSK-modulated pilot signals in wireless transmission, as this will provide valuable insights relevant to practical situations. Additionally, it is reasonable to compare our methodology with other statistical-based machine learning approaches and goodness-of-fit testing in these applications. Overall, our proposed approach demonstrates advantages in terms of integration, efficiency, and adaptability, establishing a solid foundation for future advancements in this field.

References

[1] Y. Arjoune and N. Kaabouch, "A comprehensive survey on spectrum sensing in cognitive radio networks: Recent advances, new challenges, and future research directions," *Sensors*, vol. 19, no. 1, p. 126, 2019.

[2] S. Rezaei, J. P. McKean, M. Biehl, and A. Javadpour, "DECORAS: detection and characterization of radio-astronomical sources using deep learning," *Monthly Notices of the Royal Astronomical Society*, vol. 510, no. 4, pp. 5891–5907, 2021.

[3] Z. Geng, H. Yan, J. Zhang, and D. Zhu, "Deep-learning for radar: A survey," *IEEE Access*, vol. 9, pp. 141 800–141 818, 2021.

[4] H. Ma, X. Yuan, J. Wang, and B. Li, "Revisiting model order selection: A sub-nyquist sampling blind spectrum sensing scheme," *IEEE Transactions on Wireless Communications*, pp. 1–13, 2022.

[5] G. George and C. Prema, "Cyclostationary feature detection based blind approach for spectrum sensing and classification," *Radioengineering*, vol. 27, pp. 298–303, 04 2019.

[6] S. Benazzouza, M. Ridouani, F. Salahdine, and A. Hayar, "A survey on compressive spectrum sensing for cognitive radio networks," in *2019 IEEE International Smart Cities Conference (ISC2)*, 2019, pp. 535–541.

[7] R. Ahmed, Y. Chen, B. Hassan, and L. Du, "Cr-iotnet: Machine learning based joint spectrum sensing and allocation for cognitive radio enabled iot cellular networks," *Ad Hoc Networks*, vol. 112, p. 102390, 2021.

[8] A. Mohammad, F. Awin, and E. Abdel-Raheem, "Case study of tv spectrum sensing model based on machine learning techniques," *Ain Shams Engineering Journal*, vol. 13, no. 2, p. 101540, 2022.

[9] K. Zhang, J. Li, and F. Gao, "Machine learning techniques for spectrum sensing when primary user has multiple transmit powers," in *2014 IEEE International Conference on Communication Systems, ICCS*. Macau, China: IEEE, 2014, pp. 137–141.

[10] O. P. Awe, "Machine learning algorithms for cognitive radio wireless networks," Ph.D. dissertation, Loughborough University, Loughborough, U.K., November 2015.

[11] H. Xiao, X. Zhou, and Y. Tian, "Research on wireless spectrum sensing technology based on machine learning," in *International Conference on Security, Privacy and Anonymity in Computation, Communication and Storage*. Cham, Melbourne, Australia: Springer, 2018, pp. 472–479.

[12] M. R. Vyas, D. K. Patel, and M. Lopez-Benitez, "Artificial neural network based hybrid spectrum sensing scheme for cognitive radio," in *Proc. IEEE Int. Symp. Personal, Indoor, Mobile Radio Commun.* Montreal, QC, Canada: IEEE, Oct. 2017, pp. 1–7.

[13] J. Tian, P. Cheng, Z. Chen, M. Li, H. Hu, Y. Li, and B. Vucetic, "A machine learning-enabled spectrum sensing method for OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 68, no. 11, pp. 11 374–11 378, 2019.

[14] B. Khalfi, A. Zaid, and B. Hamdaoui, "When machine learning meets compressive sampling for wideband spectrum sensing," in *2017 13th International Wireless Communications and Mobile Computing Conference (IWCMC)*. Valencia, Spain: IEEE, 2017, pp. 1120–1125.

[15] A. M. Mikaeil, B. Guo, and Z. Wang, "Machine learning to data fusion approach for cooperative spectrum sensing," in *2014 International Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery (CyberC)*. Shanghai, China: IEEE, 2014, pp. 429–434.

[16] Z. Li, W. Wu, X. Liu, and P. Qi, "Improved cooperative spectrum sensing model based on machine learning for cognitive radio networks," *IET Communications*, vol. 12, no. 19, pp. 2485–2492, 2018.

[17] J. Nikonowicz, P. Kubczak, and L. Matuszewski, "Hybrid detection based on energy and entropy analysis as a novel approach for spectrum sensing," in *2016 International Conference on Signals and Electronic Systems (ICSES)*. IEEE, 2016.

[18] ——, "Impact of the decision function on the overall performance of an energy-and entropy-based hybrid detector," in *2018 International Conference on Signals and Electronic Systems (ICSES)*, Krakow, Poland, 2018, pp. 114–117.

[19] Y. Arjoune and N. Kaabouch, "On spectrum sensing, a machine learning method for cognitive radio systems," in *2019 IEEE International Conference on Electro Information Technology (EIT)*. Brookings, SD, USA: IEEE, 2019, pp. 333–338.

[20] G. T. Reddy, M. P. K. Reddy, K. Lakshmanna, R. Kaluri, D. S. Rajput, G. Srivastava, and T. Baker, "Analysis of dimensionality reduction techniques on big data," *IEEE Access*, vol. 8, pp. 54 776–54 788, 2020.

[21] J. Nikonowicz and M. Jessa, "A novel method of blind signal detection using the distribution of the bin values of the power spectrum density and the moving average," *Digital Signal Processing*, vol. 66, pp. 18–28, 2017.

[22] B. Scheers, D. Teguig, and V. Le Nir, "Wideband spectrum sensing technique based on goodness-of-fit testing," in *2015 International Conference on Military Communications and Information Systems (ICMCIS)*, 2015, pp. 1–6.

[23] A. Goldsmith, *Wireless Communications*, 1st ed. Cambridge University Press, 2005.

[24] R. W. Heath Jr and A. Goldsmith, "Why gaussian? an introduction to the mathematics of randomness in wireless communications," *IEEE Signal Processing Magazine*, vol. 16, no. 4, pp. 65–70, 1999.

[25] B. Sklar, *Digital Communications: Fundamentals and Applications*, 2nd ed. Prentice Hall, 2001, ch. 3, pp. 53–87.

[26] W. A. Gardner, *Statistical spectral analysis: A nonprobabilistic theory*. Prentice Hall, 1995.

[27] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume II: Detection Theory*. Prentice Hall, 1998.